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Reinsurance, Taxes, and Efficiency: A Contingent Claims Model of Insurance Market Equilibrium*

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This paper presents an analytical model of underwriting capacity and insurance market equilibrium under an asymmetric corporate tax schedule characterized by incomplete tax-loss offsets. We show that reinsurance causes tax shields to be reallocated to those insurers that have the greatest capacity for utilizing them. Reinsurance is therefore used as an efficient short-run mechanism to yield the optimal allocation of tax shield benefits. In equilibrium, asymmetric taxes cause insurance prices to be actuarially unfair, and the expected return on capital invested in insurance reflects the probability of paying taxes. *Journal of Economic Literature* Classification Number: G22.

1. INTRODUCTION

The economic analysis of insurance and reinsurance markets has traditionally been modeled in an expected utility framework. In Borch (1960, 1962), risk averse insurers set up a reinsurance pool to share risks according

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to a rule derived from the first-order conditions for Pareto-optimal allocation. If these insurers all have hyperbolic absolute risk aversion (HARA) class utility functions, then reinsurance will be demanded and supplied on a proportional basis. In Blazenko (1986) and Eden and Kahane (1990), insurers and reinsurers make optimal reinsurance decisions in a mean-variance framework. In Blazenko's model, reinsurance provides additional capacity to the market, thus allowing the supply of insurance to increase, with a corresponding decline in price. In Eden and Kahane's model, the joint existence of local insurers and international reinsurers reconciles the two conflicting objectives of risk spreading and close monitoring of moral hazard.

Although the expected utility framework has provided important insights into the operation of reinsurance markets, it also has its shortcomings. Garven (1987) criticized this approach for its failure to consider the influence of competitive financial markets upon insurers' behavior and ignoring the ''nexus of contracts'' nature of the insurance firm. In the literature on the demand for corporate insurance, Main (1982, 1983) and Mayers and Smith (1982, 1990) have proposed a value-maximization approach based upon tax and agency cost considerations. A similar approach has been used by Garven (1995) to explain the demand for reinsurance by insurers holding diversified insurance portfolios. Doherty and Tiniç (1981) also criticized the assumption of risk aversion because insurance risk is largely diversifiable in the financial market. They argued that reinsurance purchases are more plausibly motivated by bankruptcy cost considerations in a value-maximization framework.

This paper analyzes the effects of taxes on underwriting capacity and equilibrium in insurance and reinsurance markets. We develop a model in which reinsurance transactions are motivated by tax considerations. The model uses an option-pricing framework.

Heaton (1986) shows that trade in leasing contracts may be motivated by the existence of incomplete state-contingent tax-loss offsets. Green and Talmor (1985) show that shareholders of levered firms underinvest in risky assets and purchase corporate insurance in order to avoid underutilizing corporate tax shields. We draw upon conceptual analogies between leasing and reinsurance markets to show that insurers' tax situations create incentives to share risks via reinsurance transactions. We show that reinsurance reallocates tax shields to those insurers that can most effectively utilize them. Like leasing, reinsurance is an economically efficient mechanism to attain an optimal allocation of tax shield benefits. We assume that taxes are *asymmetric* in that tax-loss offsets are incomplete. Since tax shields may be underutilized, the government effectively holds a portfolio of contingent claims on the taxable incomes of corporations.

Our analysis implicitly assumes that tax shield underutilization is an important problem for insurers. Casual evidence supports this assumption. During the 25 years prior to the passage of the Tax Reform Act of 1986, the U.S. property-liability insurance industry incurred on average a zero net federal income tax liability (see Walker (1991)). It is doubtful that the industry was able to consistently minimize corporate taxes without wasting valuable tax shields.

Factors other than asymmetric taxes may also motivate reinsurance. The primary requirement is that there be some cost associated with low income; consequently, bankruptcy costs or agency costs could also be substituted analytically for concave utility. In our model, insurers are assumed to be risk neutral, but the valuation function is nonlinear due to the convexity of the firm's tax liability. Therefore, by Jensen's inequality, the expected tax payment is greater than the tax on expected income. As Smith *et al.* (1990) have noted, convex tax schedules induce hedging. In our model, reinsurance provides such a hedging mechanism. We find that asymmetric taxes are *sufficient* but not *necessary* to support a reinsurance market.[1]

The paper is organized as follows. Section 2 presents a model of the insurance market in which risk neutral insurers possess a given equity capital, or surplus endowment.[2] They face an asymmetric corporate tax schedule and underwrite optimal shares in a diversified portfolio of insurance risks. In equilibrium, the optimal market shares are directly related to the relative financial capacities of the insurers. Section 3 introduces reinsurance. We argue that, given endowed shares in the insurance market, reinsurance represents an efficient mechanism to obtain optimal risk sharing. Reinsurance reallocates tax shields among insurers. The equilibrium set of reinsurance transactions minimizes the aggregate value of the government's tax claim on the income of the insurance industry.[3] Section 4 extends the model to the international reinsurance market by considering "tax clientele" effects that arise when tax rates differ among countries. Section 5 provides a long-run extension of the equilibrium argument. Incumbent firms are allowed to vary their surplus, and new firms may enter the market. This influences the allocative role of reinsurance and the after-tax equilibrium expected return on capital invested in the insurance industry. Section 6 concludes by summarizing our main results and providing suggestions for future research.

[1] Tax shield underutilization appears to be a problem for firms generally. Heaton (1987) notes that on average, anywhere from 40 to 60% of all U.S. corporations typically pay no income taxes in any given year.

[2] "Surplus" is generally used to connote "equity" in the insurance literature, and we follow this convention.

[3] Although we do not consider the effects of personal taxes, our theory parallels the capital structure theories of Miller (1977) and DeAngelo and Masulis (1980), wherein corporate debt financing is used to minimize the aggregate value of the government's tax claim.

2. A MODEL OF THE INSURANCE MARKET

2.1. *Model Assumptions and Notation*

The model has two points in time: $t = 0$, the present, and $t = 1$, the future. The economy encompasses a pool of homogeneous insurance risks with realizations occurring in the future. Aggregate claims costs, X, are normally distributed and stochastically independent of social wealth. The expected value of X is E_x, and its standard deviation is σ_x.

Let p represent the competitive market price of the economy-wide risk pool. This price is taken as given by all insurance market participants.

There are m insurers supplying insurance. At present, each insurer j ($j = 1, \ldots, m$) is endowed with surplus S_j and underwrites a fraction γ_j of the economy-wide risk pool.[4] Since this basic model involves a short-run time horizon, m and S_j are initially assumed to be fixed for all j. Moreover, transaction costs and bankruptcy risk are assumed to be negligible.

For simplicity, we assume that all insurers adopt the same investment policy of investing all their funds at $t = 0$ (surplus plus premium income) in the riskless asset with rate of return r.[5] Thus, firms differ only with respect to their endowed surplus, S, and γ is the only control variable ($\gamma \in [0, 1]$).

Claims are paid in the future. Since aggregate claims are uncorrelated with social wealth (and therefore diversifiable in the financial market), each insurer behaves as if it is risk neutral. Finally, each insurer pays taxes on the sum of underwriting profit and investment income at the rate τ. However, taxes are asymmetric in the sense that, while the government taxes gains, it does not rebate losses.[6]

As depicted in Fig. 1, the tax payment represents a put option written against the insurer's claims costs, Z. The exercise price on this option, K, consists of the sum of premium and investment income. If claims costs exceed income ($Z > K$), then the insurer does not pay taxes. However, if

[4] An important implication of this assumption is that claim payouts on insurer liabilities are perfectly correlated across all insurers. Therefore, our model does not admit the possibility of a diversification motive for reinsurance. Furthermore, as the referee correctly pointed out, if losses are perfectly correlated, then reinsurance need not be a prospective transaction. Indeed, reinsurance could be transacted retrospectively as a mechanism for exploiting differences in marginal tax rates between ceding and assuming insurers. A similar rationale has been offered in analyses of a famous retrospective liability insurance transaction that occurred in 1981 involving the MGM Grand Hotel (Las Vegas, Nevada) (e.g., see Smith and Witt (1985)).

[5] In the United States, property-liability insurers typically invest most of their assets in fixed income securities. In many other countries, insurers must invest primarily in safe assets such as government bonds.

[6] If taxes were symmetric, γ would optimally be either 0 (no insurance supply) or 1 (infinite insurance supply), depending on whether p was less or greater than its actuarially fair value.

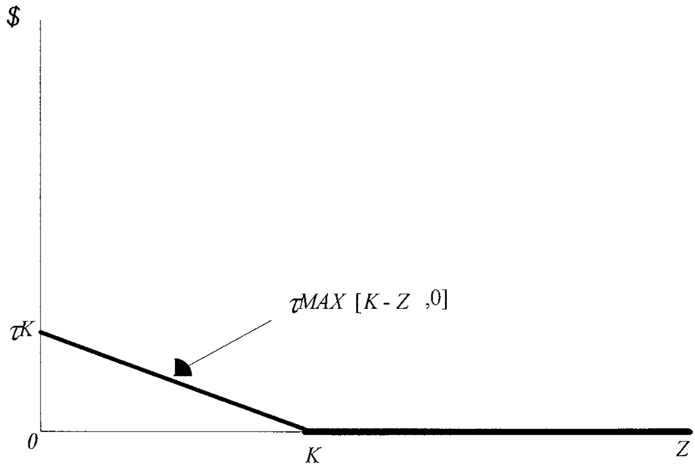

FIG. 1. Profile of tax payment.

income exceeds claims costs ($K > Z$), then the insurer pays taxes on the difference. Therefore, the insurer's tax liability is $\tau[K - Z]$, and the tax payment may be written as $\tau\text{MAX}[0, K - Z]$.

2.2. *The Model*

Since the insurer is risk neutral, management maximizes the after-tax expected present value of the firm. Ignoring the subscript j, the maximand is

$$V(\gamma) = S + \gamma p - R^{-1}\text{E}_z - \tau R^{-1}\text{E}\{\text{Max}[0, K - Z]\}, \tag{1}$$

where

$$R = 1 + r;$$
$$K = rS + \gamma pR;$$

and

$$Z = \gamma X.$$

Since the insurer underwrites a fraction γ of the economy-wide risk pool, its claims costs, Z, are normally distributed with expected value $\text{E}_z = \gamma\text{E}_x$ and standard deviation $\sigma_z = \gamma\sigma_x$.

Let $P(K, Z) = R^{-1}\text{E}\{\text{Max}\{0, K - Z]\}$ represent the value of the tax (put) option. The after-tax present value of the insurer may therefore be rewritten:

$$V(\gamma) = S + \gamma(p - R^{-1}\mathrm{E}_x) - \tau P(K, Z). \tag{1'}$$

Since Z is normally distributed, we can expand the expression for the tax option, thereby obtaining[7]

$$P(K, Z) = R^{-1}\gamma\sigma_x[d\mathrm{N}(d) + \mathrm{n}(d)], \tag{2}$$

where $\mathrm{n}(\cdot)$ is the standard normal density function, $\mathrm{N}(\cdot)$ is the standard normal distribution function, and d is

$$d = \frac{K - \mathrm{E}_z}{\sigma_z} = \frac{rS + \gamma(pR - \mathrm{E}_x)}{\gamma\sigma_x}. \tag{3}$$

Next, we calculate the first and second derivatives of P with respect to γ.

$$\begin{aligned}\frac{\partial P}{\partial \gamma} &= R^{-1}\sigma_x\left[\mathrm{N}(d)\left(d + \gamma\frac{\partial d}{\partial \gamma}\right) + \mathrm{n}(d)\right] \\ &= R^{-1}\left[\mathrm{N}(d)(pR - \mathrm{E}_x) + \sigma_x\mathrm{n}(d)\right],\end{aligned} \tag{4}$$

and

$$\frac{\partial^2 P}{\partial \gamma^2} = -\frac{rS}{\gamma}R^{-1}\mathrm{n}(d)\frac{\partial d}{\partial \gamma} > 0. \tag{5}$$

From Eqs. (1′) and (5), we obtain the second-order condition (SOC)

$$V''(\gamma) = -\tau(\partial^2 P/\partial\gamma^2) < 0.$$

Consequently, the after-tax present value of the firm, V, is concave in the decision variable γ. From Eq. (4), the put value increases in γ when $pR \geq \mathrm{E}_x$, that is, irrespective of whether insurance is actuarially fair or unfair.

Assuming that an interior solution exists, from Eqs. (1′) and (4) we obtain the first-order condition (FOC)

$$(pR - \mathrm{E}_x)\,[1 - \tau\mathrm{N}(d)] = \tau\sigma_x\mathrm{n}(d). \tag{6}$$

[7] Note that this expression is similar to Brennan's (1979) Eq. (39) for the discrete-time-neutral valuation of an option when the return distribution of the underlying asset is normal.

The left-hand side of Eq. (6) represents the after-tax expected marginal gain from an increase in γ. The right-hand side represents the marginal tax loss from the increased probability that in the future the put option will be in the money.

Solving Eq. (6) for p,

$$p = R^{-1}\{\mathrm{E}_x + \lambda(\gamma)\sigma_x\}. \tag{7}$$

In Eq. (7), the function $\lambda(\gamma) = (\tau\, \mathrm{n}(d))/(1 - \tau \mathrm{N}(d))$ is positive and represents the unit risk loading factor.

In our model, the insurer makes an optimal underwriting decision by appropriately adjusting γ, thereby obtaining (at the firm level) an actuarially unfair price for insurance (for given values of p, E_x, σ_x, and R). If insurance is actuarially fair (i.e., if $p = R^{-1}\mathrm{E}_x$), then the FOC implies that $\mathrm{n}(d)$ must equal zero. Hence, d tends toward infinity. Since $d = rS/\gamma\sigma_x$, this further implies that $\gamma = 0$; hence, no insurance is supplied. For positive values of γ, the FOC is satisfied only if insurance is actuarially unfair (i.e., if $p > R^{-1}\mathrm{E}_x$). Finally, note that if there are no taxes, i.e., if $\tau = 0$, then equilibrium insurance prices are actuarially fair.[8]

2.3. *Comparative Statics*

LEMMA 1. The optimal insurance supply increases in S, p, and r, and decreases in τ and σ_x.

Proof. We show the proof for surplus (S) only.[9] Differentiating implicitly from the FOC with respect to γ and S, we obtain

$$\frac{\partial \gamma}{\partial S} = -\frac{\partial V'/\partial S}{V''(\gamma)}.$$

Since the SOC is satisfied (i.e., $V''(\gamma) < 0$), this implies that $\mathrm{sign}(\partial\gamma/\partial S) = \mathrm{sign}(\partial V'/\partial S)$. Now, $\partial V'/\partial S = -\tau(\partial^2 P/\partial\gamma\partial S)$. Differentiating the expression for $\partial P/\partial\gamma$ in Eq. (4) with respect to S,

$$\partial V'/\partial S = \tau\, \mathrm{n}(d)\, r^2 S/R\gamma^2\sigma_x > 0. \tag{8}$$

Hence $\partial\gamma/\partial S > 0$. Q.E.D.

[8] The intuition that competitively determined insurance prices must incorporate a corporate tax loading is a well-known result from the insurance pricing literature. See Fairley (1979), Hill (1979), Doherty and Garven (1986), Hill and Modigliani (1987), and Myers and Cohn (1987).

[9] The complete set of proofs showing how parameter changes influence optimal underwriting are available upon request from either author.

2.4. *Market Equilibrium*

The market equilibrium condition is written

$$\Sigma_j \gamma_j = 1. \tag{9}$$

The FOC (see Eq. (6)) defines the optimal market share γ as a function of p, E_x, r, τ, σ_x, and S. From the proof of Lemma 1, using Eqs. (5) and (8) and the expression for $V''(\gamma)$, we obtain $\partial\gamma/\partial S = \gamma/S$. Hence, γ is linear in S. The optimal value for γ_j, insurer j's market share, may thus be written as

$$\gamma_j = S_j \cdot h(p, \tau, \sigma_x, \mathrm{E}_x, r).$$

This follows from the fact that insurers differ only with respect to their endowed surplus. Substituting this expression into Eq. (9), we obtain

$$\Sigma_j \gamma_j = h(\cdot)\, \Sigma_j S_j = 1.$$

Consequently, $h(\cdot) = 1/\Sigma_j S_j$, and

$$\gamma_j = S_j/\Sigma_j S_j = s_j, \tag{10}$$

where s_j represents insurer j's share of total industry surplus.

Equation (10) indicates an optimal sharing rule that we formally define in the following proposition.

PROPOSITION 1. *In equilibrium, the share of insurer j in the insurance market is equal to its share in the industry's surplus*: $\gamma_j = s_j$.

Given price-taking behavior, the loading factor λ introduced in Eq. (7) should be the same across all insurers. However, the value of λ is firm-specific, since it depends on d, which in turn depends on γ. This is no longer true when the market is in equilibrium. For insurer j,

$$d_j = \frac{rS_j + \gamma_j\,(pR - \mathrm{E}_x)}{\gamma_j \sigma_x}.$$

In equilibrium, $S_j = \gamma_j \Sigma_j S_j$. Hence, we obtain

$$d_j = \frac{r\Sigma_j S_j + pR - \mathrm{E}_x}{\sigma_x},$$

which is firm-independent.

The dynamics of equilibrium may be described as follows: If $\Sigma_j \gamma_j < 1$, there is an excess demand in the market. The excess demand drives up the price (the loading). Therefore, the optimal value for γ is increased for all firms. From (3), the change in d is the joint result of the increases in p and γ:

$$\frac{\mathrm{d}d}{\mathrm{d}p} = \frac{\partial d}{\partial p} + \frac{\partial d}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial p} = -\frac{R\gamma}{\lambda r S} < 0.$$

Thus d decreases, but since $\partial\lambda/\partial d < 0$, the increased loading is validated in Eq. (7). The excess demand eventually vanishes.

3. REINSURANCE AND EFFICIENCY

In the model presented in the previous section, market equilibrium implies a precise distribution of demand across insurers. Each insurer takes a share in total demand corresponding exactly to that insurer's share in total industry surplus.

However, since insurers are assumed to be identical in all respects except for surplus endowments, and since bankruptcy risk has been assumed away, consumers are indifferent with respect to the choice of an insurer. Consumers will be distributed randomly among insurers, and it would be coincidental if the *ex ante* distribution of consumers would replicate the optimal allocation of risks among insurers. Therefore, the equilibrium process not only implies the usual adjustment of supply and demand, but also a reallocation of consumers among insurers. Those insurers facing an excess demand at the equilibrium price deny coverage to some consumers who must find an insurer with excess capacity.

In practice, such a reallocation would impose costs ignored in our model, such as the marketing costs of attracting additional consumers, or the agency costs (e.g., loss of goodwill) associated with refusing coverage to some consumers.

Insurers could be given the opportunity to adjust their optimal supply to the observed distribution of consumers. Surplus provides the key to this solution. From Eq. (10), optimal risk sharing obtains either by adjusting γ for a given value of S, or by proceeding the other way around. However, this would not be easily implemented in practice. Surplus cannot be adjusted in the short run without incurring transactions and/or agency costs.

The insurance market provides a simpler and more efficient method of consumer reallocation, namely, reinsurance. This mechanism may be introduced into our model by redefining γ as *net* underwriting. Let α_j represent the endowed market share of the jth insurer ($\alpha_j > 0$, $j = 1, \dots,$

m) and let β_j represent the fraction of the market which insurer j reinsures ($\beta_j \leq \alpha_j$).[10] If $\beta_j > 0$, the insurer *cedes* a part of his insurance portfolio by purchasing reinsurance. If $\beta_j < 0$, the insurer *assumes* reinsurance. Net underwriting is thus $\gamma_j = \alpha_j - \beta_j$, where β_j represents the new control variable. Dropping the subscript j, the after-tax present value of the firm may be restated as

$$V(\beta) = S + (\alpha - \beta)(p - R^{-1}\mathrm{E}_x) - \tau P(K, Z), \tag{11}$$

where

$$K = rS + (\alpha - \beta)pR$$

and

$$Z = (\alpha - \beta)X.$$

Note that the *law of one price* guarantees the same price for the portfolio of risks offered in both the insurance and the reinsurance markets. If the reinsurance price exceeded the insurance price, insurers would have an incentive to withdraw from the insurance market and specialize in supplying reinsurance. Therefore (consistent with observed practice), we assume that quota-share reinsurance occurs on "original terms."

Proceeding from Eq. (11) as we did in the previous section, we obtain Eqs. (2), (3), (6), and (7), where γ is simply replaced by $\alpha - \beta$. Thus the earlier result that equilibrium insurance prices are actuarially unfair and include a loading for taxes still holds.

The market equilibrium conditions, $\Sigma_j\beta_j = 0$ and $\Sigma_j\alpha_j = 1$, combined with the comparative statics results,

$$\frac{\partial\beta}{\partial\alpha} = 1 \qquad \text{and} \qquad \frac{\partial\beta}{\partial S} = -\frac{\alpha - \beta}{S},$$

yield

$$\beta_j = \alpha_j - s_j. \tag{12}$$

Equation (12) is essentially a restatement of Eq. (10), which defined Proposition 1's optimal sharing rule.

However, Eq. (12) provides an additional dimension. It implies that in

[10] This is quota-share reinsurance. The quota-share is defined as $q_j = \beta_j/\alpha_j$.

equilibrium, given α, insurers with high surplus endowments will tend to reinsure insurers with low surplus endowments. In particular, if all insurers underwrite the same share of the insurance market, i.e., if $\alpha_j = 1/m$ for all j, we obtain $\beta_j = (1/m) - s_j$. Defining the average surplus as $\overline{S} = \Sigma_j S_j/m$, and the average surplus share as $\bar{s} = \overline{S}/\Sigma_j S_j$, we obtain $\bar{s} = 1/m$. Hence,

$$\beta_j = \bar{s} - s_j. \tag{13}$$

In equilibrium, with identical *ex ante* direct market shares, insurers with higher than average surplus endowments assume reinsurance, whereas insurers with lower than average surplus endowments purchase reinsurance. This is stated as the following.

PROPOSITION 2. *In equilibrium, given* α, *high surplus firms reinsure low surplus firms.*

It is instructive to link Proposition 2 to the tax feature of our model. As implied by the FOC (see Eq. (6)), the insurer's determination of the optimal level of reinsurance involves a tradeoff. Increasing γ by supplying reinsurance increases the insurer's risk. Consequently, the value of the tax option increases (see Eq. (4)). But this also increases the expected return from underwriting. For a given value of γ, if S is larger, then K is also larger. Hence, the tax put is more deeply in the money and therefore more valuable. However, in this case, the asymmetric nature of the tax liability is not as important as before. Compared with poorly capitalized insurers, well-capitalized insurers incur less risk of not being able to fully utilize tax shields. This is reflected in $\partial^2 P/\partial\gamma\partial S < 0$.[11] The sensitivity of the put value to changes in γ is smaller for larger values of S. Consequently, the tradeoff is more attractive for the better capitalized insurers; they are able to assume more insurance risk.

In our model, reinsurance reallocates risks among insurers due to incentives arising from asymmetries in the tax system. This contrasts with the traditional motivation in the insurance literature, where the purpose of reinsurance is to enable insurers to construct more balanced portfolios of net underwriting.

Our reinsurance model presents interesting analogies with the work of Karl Borch:

- In Borch (1962), insurers are endowed with insurance portfolios. They are risk averse and use reinsurance to reallocate insurance risks among themselves. A Pareto-optimal sharing of risk occurs when all insurers participate in a reinsurance pool in proportion to their respective degrees of risk tolerance. In contrast, our insurers are risk neutral, and market equilibrium

[11] Note that from Eq. (8), $\partial^2 P/\partial\gamma\, \partial S = -\mathrm{n}(d) r^2 S/R\gamma^2\sigma_x$.

results in an optimal sharing rule that is related to the endowed financial capacity of each insurer rather than risk aversion.

- In Borch (1985), insurers are also endowed with insurance portfolios, but they are assumed to be risk neutral. They demand reinsurance because they must comply with solvency constraints, and reinsurance is less costly than raising capital in the financial market. There is a clear analogy with our model. Reinsurance and surplus are substitutes for reaching a given objective: the solvency constraint in Borch's model, and optimal allocation of tax shields in our model. In both cases reinsurance is used because it is a more efficient mechanism.

Note that the second analogy indicates that reinsurance is a capital structure decision. By adjusting reinsurance demand or supply, each insurer selects its net amount of insurance leverage for a given level of surplus.[12] An increase in reinsurance demand means that net liabilities are lower, i.e., leverage decreases. Thus, our model provides a tax motivation for reinsurance that parallels tax-based theories of optimal capital structure in corporate finance.

Before we extend our model further to an international setting, consider a tax-based interpretation of the traditional view that emphasizes the risk reducing aspects of reinsurance. By assuming that insurers share in a diversified portfolio of risks, we obviously set aside this motivation at the outset. However, suppose that claim payouts on insurer liabilities are uncorrelated. Therefore, by arranging reciprocal reinsurance agreements, insurers could reduce risk and, given convexity in tax schedules, consequently reduce expected tax liabilities. Under this alternative story, σ_x would be reduced, γ remaining constant. Assuming a constant value for E_x, Lemma 1 shows that both insurers would then optimally react by increasing γ. In equilibrium, the insurance price would be lower if all firms were able to reduce σ_x by using reciprocal reinsurance.[13] However, our market equilibrium result would still hold. Our model applies to an economy where all valuable forms of risk reducing reciprocal reinsurance have already been fully utilized.

4. INTERNATIONAL REINSURANCE AND TAXES

Insurance transactions typically occur domestically, whereas reinsurance is often transacted internationally. This is mainly due to nontariff barriers

[12] Note that in equilibrium, leverage is equalized across all firms. Defining leverage as $l_j = \gamma_j p/S_j$, and using the relation $\gamma_j = s_j$, we find that $l_j = p/\Sigma_j S_j$ for all j.

[13] From Eq. (7), $dp/d\sigma_x = R^{-1}\{\lambda + \sigma_x(\partial\lambda/\partial d)(\partial d/\partial\sigma_x)\}$. This expression is positive since both $(\partial\lambda/\partial d)$ and $(\partial d/\partial\sigma_x)$ are negative. Therefore, by reducing risk, reinsurance causes insurance prices to decline.

to trade in services. Consumer protection is typically invoked as a justification for protectionism in the provision of insurance services.[14] However, this argument cannot be used to erect barriers to reinsurance trade. Moreover, freedom in international reinsurance trade is generally considered an important requirement for a sufficient spreading of risks.

Given protection in the provision of insurance services, our model provides an additional explanation for international reinsurance trade. As may be expected from Lemma 1, other things equal, insurers in low-tax countries will tend to provide reinsurance to insurers in high-tax countries. In this section, we provide a formal proof of this implication by extending our model to two countries: the domestic country and a foreign country.

In the following, variables for the foreign country are denoted by an asterisk, and foreign (re)insurance firms are indexed by k, $k = 1, \dots, m^*$. The model of Section 3 is assumed to apply in both countries; but some specific assumptions are necessary to substantiate the *ceteris paribus* assumption and to emphasize the role of taxes. These assumptions are

A1 No foreign exchange risk. The exchange rate is fixed and equals unity.

A2 Capital flows across countries are free.

A3 Insurance markets are segmented by nontariff barriers, but international reinsurance trade is free.

A4 Same insurance risk in both countries: $\mathrm{E}_x = \mathrm{E}_x^*$ and $\sigma_x = \sigma_x^*$.

A5 All firms have the same surplus: $S_j = S_k^* \equiv S$ for all j and k.

A6 The tax rates differ in the two countries, $\tau > \tau^*$.

A7 No arbitrage opportunities.

Assumptions A1, A2, and A7 together imply that $r = r^*$. Assumptions A3, A4, and A7 together imply that $p = p^*$.

Applying the optimal value of γ from Section 2 to firms in each country, we obtain

$$\gamma_j = \alpha_j - \beta_j = S_j \cdot h\,(p, \tau, \sigma_x, E_x, r), \qquad j = 1, \dots, m$$
$$\gamma_k^* = \alpha_k^* - \beta_k^* = S_k^* \cdot h(p^*, \tau^*, \sigma_x^*, \mathrm{E}_x^*, r^*), \qquad k = 1, \dots, m^*.$$

Given our assumptions, this may be written more compactly as

$$\gamma_j = \alpha_j - \beta_j = S \cdot h(\tau), \qquad \text{for all } j, \tag{14a}$$

and

[14] See, for example, Skipper (1987) and Outreville (1989). Note, however, that liberalization of trade in services, including insurance, represented a major issue in the "Uruguay Round" negotiations at GATT.

$$\gamma_k^* = \alpha_k^* - \beta_k^* = S \cdot h(\tau^*), \qquad \text{for all } k. \tag{14b}$$

Lemma 1 and A6 together imply $h(\tau) < h(\tau^*)$. Therefore,

$$\gamma_j < \gamma_k^*, \qquad \text{all } j \text{ and } k. \tag{15}$$

Firms in the low-tax country supply more net insurance than firms in the high-tax country.

The market equilibrium conditions are now written as

$$\text{insurance: } \sum_j \alpha_j = \sum_k \alpha_k^* = 1 \tag{16}$$

$$\text{reinsurance: } \sum_j \beta_j + \sum_k \beta_k^* = 0. \tag{17}$$

Condition (16) reflects the segmentation of insurance markets, whereas condition (17) reflects the extension of our model to a two-country setting where there are no barriers to reinsurance trade.

We are now ready to prove the next proposition.

PROPOSITION 3. *In equilibrium, given α, firms in the low-tax country reinsure firms in the high-tax country.*

Proof. Assume $\alpha_j = \alpha_k^* = \alpha$ for all j and k (From (16), this implies $m = m^*$). Using (14a) and (14b) we obtain

$$\beta_j = \alpha - S \cdot h(\tau) = \beta, \qquad \text{for all } j,$$

and

$$\beta_k^* = \alpha - S \cdot h(\tau^*) = \beta^*, \qquad \text{for all } k.$$

Since $h(\tau) < h(\tau^*)$, we have $\beta > \beta^*$. Furthermore, since (17) is now written $m\beta + m\beta^* = 0$, these two conditions together imply that $\beta > 0$ and $\beta^* < 0$. Q.E.D.

Reinsurance enables insurers to circumvent international insurance trade barriers. This is particularly important in an environment characterized by differential tax treatment of insurer income across countries, because taxes create underwriting capacity constraints. Consequently, the underwriting capacity of local insurers in high-tax domiciles is augmented by the opportunity to cede risks to reinsurers operating in low-tax domiciles, as the latter are willing to assume more risk. This helps to explain why large reinsurers tend to be located in certain countries (e.g., Switzerland and Germany), and why captive insurers tend to be formed in low-tax domiciles such as Bermuda and the Cayman Islands.

Note, however, that Proposition 3 is derived under the assumption of identical endowed market shares for all firms in both countries. This is in the spirit of our assumption of identical firms. However, as is clear from inequality (15), an insurer in the high-tax country could provide reinsurance services to an insurer in the low-tax country, so long as α_k^* is sufficiently larger than α_j.

An empirical implication of Proposition 3 is that average net retention ratios should be inversely related to corporate tax rates. Empirical support for Proposition 3 is provided by Outreville (1994), who finds a statistically significant *negative* relationship between retention ratios and corporate tax rates in 42 developing countries during 1988 and 1989.

5. SURPLUS AND THE EQUILIBRIUM AFTER-TAX RETURN

Thus far, we have limited our analysis to the short run and argued that it is more efficient for insurers to use reinsurance rather than capital in order to make adjustments toward their equilibrium market share. However, in the long run, some insurers may realize that they are undercapitalized. This will happen if they must regularly cede a fraction of their insurance portfolio, thereby foregoing positive expected profits. Such insurers will face incentives to raise more capital. The cost of raising additional capital will be balanced by the additional expected profits over several periods of time. Consequently, more capital will be raised by some firms in the long run. The industry's total surplus will increase, and reinsurance will play a less important role in the optimal allocation of tax shields among firms.[15]

Previously, we also considered the number of insurers as given and equal to m. However, since insurance is actuarially unfair, there is an incentive for risk neutral investors to enter the insurance market, thereby increasing the total surplus of the insurance industry. The combination of low surplus and a higher than average market share will create incentives for these firms to cede reinsurance.

Let $S^* = \Sigma_j S_j$ represent the total surplus of the insurance industry. In Section 2.4, we noted that S^* enters into the equilibrium expression for d:

$$d = \frac{rS^* + pR - \mathrm{E}_x}{\sigma_x}. \tag{18}$$

From Eq. (7), we obtain

[15] Furthermore, the firms with excess capacity will be able to increase their marketing effort in the long run.

$$\frac{pR - \mathrm{E}_x}{\sigma_x} = \frac{\tau \mathrm{n}(d)}{1 - \tau \mathrm{N}(d)} = \lambda(\tau, d), \tag{19}$$

where λ represents the unit-risk loading factor.[16] Substituting (19) into (18), we obtain a new equilibrium expression for d:

$$d = \frac{rS^*}{\sigma_x} + \lambda(\tau, d). \tag{20}$$

Next, define $\mathrm{E}(r_i)$ as the after-tax expected rate of return on capital invested in an insurance firm. By definition,

$$\mathrm{E}(r_i) = S^{-1}\{RS + \gamma(pR - \mathrm{E}_x) - \tau \mathrm{E}[\mathrm{Max}(0, K - Z)] - S\}.$$

Using Eqs. (2), (7), (10), and (20), we derive the equilibrium value for $\mathrm{E}(r_i)$:

$$\mathrm{E}(r_i) = r[1 - \tau \mathrm{N}(d)]. \tag{21}$$

In equilibrium, the after-tax expected rate of return on insurance is equal to the after-tax rate of return on the riskless asset, adjusted for the probability of paying taxes.[17] Since $\mathrm{N}(d)$ is less than one, it follows that the net expected return on insurance is larger than the net return on a riskless investment. This result obtains in spite of our earlier assumption that insurance underwriting has no systematic risk. Consequently, asymmetric taxes result in a reward for idiosyncratic risk. The additional return is equal to the expected tax payment, adjusted for the probability that losses are sustained:

$$\mathrm{E}(r_i) - r(1 - \tau) = \tau r(1 - \mathrm{N}(d)).$$

LEMMA 2. *The equilibrium value of d is increasing in S^*.*

Proof. From Eq. (20), taking r, τ, and σ_x as given, the equilibrium value of d is defined by an implicit function:

$$\mathrm{F}(d, S^*) = d - \frac{rS^*}{\sigma_x} - \lambda(\tau, d) = 0.$$

[16] Note that this loading factor represents an application of Sharpe's reward-to-variability ratio to insurance underwriting.

[17] $\mathrm{N}(d)$ corresponds to the probability that taxes are paid by the insurance industry as a whole. In terms of the notation used here, $\mathrm{N}(d)$ corresponds to the probability that $X < K$, where $K = rS^* + pR$ (see Eq. (18)).

When S^* increases, the total change in d from one equilibrium state to the other is

$$\frac{\mathrm{d}d}{\mathrm{d}S^*} = -\frac{\partial \mathrm{F}/\partial S^*}{\partial \mathrm{F}/\partial d} = \frac{r}{\sigma_x}\left(1 - \frac{\partial \lambda}{\partial d}\right)^{-1}.$$

Since $\partial\lambda/\partial d < 0$, $\mathrm{d}d/\mathrm{d}S^* > 0$. Q.E.D.

Lemma 2 implies that when additional capital flows into the insurance industry, $\mathrm{N}(d)$ increases, thereby causing the value of $\mathrm{E}(r_i)$ to decline (see Eq. (21)). This occurs either because some firms increase their surplus, new firms enter the market, or both. Classically, long-run equilibrium obtains when the after-tax expected return on insurance is equal to the after-tax expected return on the other activities with no systematic risk.

The preceding results may be summarized in the following proposition.

PROPOSITION 4. *The equilibrium after-tax expected return on insurance decreases when additional capital flows into this industry. In a long-run equilibrium, this return is equal to the after-tax expected return on activities with no systematic risk. Idiosyncratic risk is rewarded by an excess return depending on the tax rate and on the probability that the insurance business generates losses.*

6. SUMMARY AND CONCLUSION

We have utilized a contingent-claims framework to investigate the pricing and incentive effects of asymmetric taxes on net insurance supply. Our analysis makes three contributions to the theory of insurance markets.

Our first contribution relates to underwriting capacity. Given risk neutrality and the absence of bankruptcy risk, market equilibrium is characterized by the existence of actuarially unfair prices. In this equilibrium, the optimal sharing rule is defined by the relative financial capacity of each insurer.

Our second contribution relates to the theory of reinsurance. In the short run, asymmetric taxes constitute a sufficient condition to motivate reinsurance transactions. Reinsurance is a mechanism for *efficiently* allocating tax shields among risk neutral insurers. We found that well-capitalized firms have a comparative advantage in providing reinsurance services, as do firms that are domiciled in countries with low tax rates. These predictions are consistent with stylized facts about the reinsurance market.

Since reinsurance is a capital structure decision, these results parallel those obtained in the capital structure theories of Miller (1977) and DeAngelo-Masulis (1980). In equilibrium, reinsurance contracts are chosen to minimize the government's tax claims.

Our reinsurance model is also related to Green and Talmor's (1985) work on the incentive effect of asymmetric corporate taxes. In their paper, the tax liability is represented by the value of a call option on the firm's end-of-period assets. The convexity of this liability motivates underinvestment in risky projects, conglomerate mergers, and the purchase of corporate insurance. By introducing loss carryback and carryforward provisions, the incentive effects are somewhat mitigated, but the basic argument remains since future gains and losses are uncertain and must be discounted. In our paper, the tax liability is represented by the value of a put option on the firm's end-of-period claims payments. The convexity of this liability provides an incentive to purchase reinsurance. However, in contrast to Green and Talmor (1985), we add a market equilibrium dimension. This yields an optimal allocation of tax shields (expected claims payments) among firms. In equilibrium, the value of the government's tax claims on the insurance industry as a whole is minimized.

The third contribution of our paper relates to capital flows into the insurance industry. We find that the equilibrium after-tax expected return on insurance is equal to the after-tax return on the riskless asset adjusted for the probability of insurers paying taxes. This expected return is decreasing in the total capital invested in the industry. In the long run, some insurers will increase their surplus as a substitute for reinsurance purchases. New insurers will also enter the market if warranted by the equilibrium after-tax expected return on insurance activity. These two effects will increase the total capital invested in the industry and thus lower its expected rate of return. Long-run equilibrium obtains when the after-tax expected returns are equal across sectors with no systematic risk.

A promising avenue for future research would be to explicitly model the joint determination of reinsurance supply, marketing effort, and changes in surplus. This would imply extending our model in three directions: first, a multiperiod framework; second, introduction of a random process for insurance demand; and third, explicit consideration of transaction costs.

Another promising avenue for future research would be to formulate either cross-sectional or time-series empirical tests that distinguish among the motives for insurance. A cross-sectional test could examine tax regimes in different countries and investigate how reinsurance flows are related to these differences. A time-series test could investigate how changes in tax regimes in one or more countries affect reinsurance. Preliminary empirical support for our theory has been provided by Outreville (1994) who shows that net retention ratios are inversely related to corporate tax rates.

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