The Demand for Reinsurance: Theory and Empirical Tests

by

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May 2003
Forthcoming in *Assurances*, Vol. 7, No. 3 (July 2003)

ABSTRACT. This paper investigates the valuation effects of reinsurance purchases in a contingent claims framework. The comparative statics of the model suggest that, other things equal, the demand for reinsurance will be greater, 1) the higher the firm's leverage, 2) the lower the correlation between the firm's investment returns and claims costs, 3) for firms which write "longer tail" lines of insurance, and 4) the more the firm concentrates its investments in tax-favored assets. These predictions are tested in an empirical analysis of the reinsurance behavior of U.S. property-liability insurance firms during the period 1980-1987.

‡**Acknowledgments.** Earlier versions of this paper benefited substantially from helpful comments offered by a number of colleagues, including Pat Brockett, David Cummins, Neil Doherty, Henri Loubergé, and seminar participants at Baylor University, Louisiana State University, The University of Pennsylvania, the American Risk and Insurance Association, and the International Financial Management Association. The authors gratefully acknowledge the support of the Gus S. Wortham Chair at the University of Texas at Austin in making the A. M. Best data available for use in this study.

Dedicated to the memory of our mentor and friend, Bob Witt.

THE DEMAND FOR REINSURANCE: THEORY AND EMPIRICAL TESTS

1. INTRODUCTION

Like most business firms, insurance companies rely upon a variety of forms of financing. However, insurance companies are “different” because they create explicit liabilities whenever they sell their products.[1] Indeed, its policyholders hold most of a typical insurer's liabilities. An insurer's success depends not only on charging adequate rates to cover costs, but also on providing credible assurances to policyholders that claim payments will be made. This is a consequence of the nature of the insurance business, since a policy pays off in the joint contingency that the insured loss event occurs and the insurance company is financially solvent. Such assurances can be credibly provided by a number of mechanisms, including the commitment of adequate equity capital, or surplus, and the purchase of reinsurance.

Reinsurance constitutes a type of insurance that involves acceptance by an insurer, called the reinsurer, of all or a part of the risk of loss covered by another insurer, called the ceding company. When an insurer cedes reinsurance to a reinsurer, the ceding firm simultaneously reduces the variability of its cash flows and its financial leverage. Therefore, the decision to reinsure can be viewed as both a risk management and a capital structure decision.

A number of alternative motivations and analytic approaches to studying the demand for reinsurance have been previously suggested in the literature. Traditionally, the analysis of the demand for reinsurance has been modeled in an expected utility framework and has primarily emphasized the risk management aspect of the reinsurance decision. In Borch (1960, 1962), risk-

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[1]This characteristic (i.e., creating contingent liabilities as a consequence of marketing products) is not unique to insurance; indeed, it is a characteristic that is common to most financial institutions. For example, a similar claim can be made about bank deposits. Although a number of factors can be cited for why risk management decisions and firm financing decisions

averse insurers set up a reinsurance pool to share risks according to a rule derived from the first-order conditions for Pareto-optimal allocation. Borch shows that if insurers have hyperbolic absolute risk aversion (HARA) class utility functions, then reinsurance will be demanded and supplied on a proportional basis. In Blazenko (1986) and Eden and Kahane (1990), reinsurance decisions are made in a mean-variance framework. In Blazenko's model, reinsurance provides additional capacity to the market by facilitating the spreading of risk. In Eden and Kahane's model, the joint existence of local insurers and international reinsurers reconciles the two conflicting objectives of risk spreading and close monitoring of moral hazard. Jean-Baptiste and Santomero (2000), Winton (1995), and Cutler and Zeckhauser (1999) address the problem of adverse selection in the market for reinsurance. Jean-Baptiste and Santomero's model highlights the role played by long-term contracting relationships in controlling adverse selection. In Winton's model, high state verification costs favor nonproportional contracts, whereas low state verification costs favor proportional contracts. However, since long-term contracting relationships between ceding insurers and reinsurers ameliorate state verification costs, Winton's model predicts that at the margin, such relationships favor proportional contracts. Similarly, Cutler and Zeckhauser find that adverse selection leads to nonlinear risk sharing in catastrophe reinsurance markets.

Other studies have analyzed the demand for reinsurance from a capital structure perspective. Doherty and Tiniç (1981) show that reinsurance is irrelevant if the pricing of insurance is inelastic with respect to the insurer's ruin probability. However, if insurance prices are sensitive to default risk, then policyholders pay a lower (higher) price for a policy when the probability of default on the part of the insurer is high (low).[2] Mayers and Smith (1981) suggest

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are interdependent (e.g., see Doherty (2000), especially chapter 8), the linkage between these decisions is even more apparent for financial institutions than it is for nonfinancial institutions.

[2]Although Doherty and Tiniç do not employ contingent claims analysis, models developed subsequently by Doherty and Garven (1986) and Cummins (1988) confirmed the implications of their model from a contingent claims perspective. Contingent claims models view risky insurance as the economic equivalent of safe insurance minus the value of a "limited

that agency costs may also be important. Like lenders in the bond market, policyholders face incentive problems in the insurance market. These incentive problems arise because shareholders may be able to effect wealth transfers between policyholders and themselves by altering various aspects of the firm's investment, underwriting or dividend policies after issuing insurance. However, since policyholders recognize the incentives faced by shareholders, the prices they are willing to pay for the policies should reflect unbiased estimates of the expected behavior of stockholders. Furthermore, the greater the firm's leverage, the greater will be the magnitude of these agency costs borne by shareholders.[3] Since the purchase of reinsurance effectively unlevers the firm, it also reduces agency costs that would otherwise be borne by shareholders in the guise of lower insurance premiums. Garven (1987) argues that in order for insurer capital structure decisions (including the decision to reinsure) to "matter" in any meaningful sense, factors such as underutilized tax-shields and costs related to financial distress (such as agency and bankruptcy costs) must be considered.[4]

This paper sets forth a theory of the demand for reinsurance in which reinsurance is characterized as both a leverage management and risk management mechanism.[5] A contingent claims framework is adopted, since this makes it possible to explicitly model the impact that reinsurance has upon the various leverage-related costs that the firm may incur. The paper is organized in the following manner. In Section 2, a valuation model is presented which considers

liability" put option (see Garven (1992)). Furthermore, several empirical studies (e.g., see Cummins and Danzon (1997), Guo and Winter (1997), Phillips, Cummins, and Allen (1998) and Sommer (1996)) lend empirical support to this notion.

[3]Garven and Pottier (1995) provide a formal analysis of the role played by participating insurance policies in the resolution of a particularly important form of stockholder opportunism: the risk shifting problem. They also provide empirical evidence that highly levered insurers have a greater propensity to issue participating insurance policies than do insurers with more conservative capital structures

[4]Hoerger, Sloan, and Hassan (1990) present a model in which the primary purpose of reinsurance is to lower the risk of bankruptcy. Empirically, they find that the level of reinsurance varies directly with claims volatility and inversely with firm size and surplus-premium ratios. Guo and Winter (1997) obtain conceptually similar theoretical and empirical results in their study of insurer capital structure equilibrium; specifically, insurer leverage is found to be significantly negatively related to the degree of uncertainty in insurance losses.

the various costs and benefits of reinsurance purchases in a contingent claims framework. Section 3 outlines various testable hypotheses that are yielded from the comparative statics of the model. Section 4 presents evidence from an empirical study of the reinsurance behavior of U.S. property-liability insurance firms during the period 1980-1987. Section 5 concludes.

2. VALUATION EFFECTS OF REINSURANCE PURCHASE DECISIONS

Next, a single-period valuation model is developed which provides the basis for a formal analysis of the demand for reinsurance. The firm is formed at the beginning of the period for the purpose of maximizing the after-tax market value of its equity, or surplus. The firm receives premium income from issuing a portfolio of homogeneous insurance policies, and purchases quota share, or proportional reinsurance on this portfolio.[6] It invests its initial surplus and net premiums in an asset portfolio that comprises various financial instruments. At the end of the period, the firm's cash flows from its investment, underwriting, and reinsurance activities are realized. Since we are primarily concerned with the valuation implications of the reinsurance decision *per se*, we treat the insurer's direct underwriting, investment and capitalization decisions are treated as given and focus upon the reinsurance decision.[7]

Our analytic approach follows a rich tradition in the financial economics literature, going back to the seminal paper on option pricing by Black and Scholes (1973). Besides deriving a closed form solution for the price of a European call option, Black and Scholes suggest that the

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[5]Although moral hazard and adverse selection issues discussed in the preceding literature review are not without interest, such issues are beyond the scope of this paper. Interested readers are referred to Garven and Lamm-Tennant's (2003) empirical analysis of the Jean-Baptiste and Santomero (2000) adverse selection model.

[6]In practice, nonproportional as well as proportional contracts are important features of the reinsurance market. Domestic reinsurance is often transacted on a nonproportional basis, whereas international reinsurance is more commonly transacted on a proportional basis. Since we are not concerned with modeling the effects of moral hazard and adverse selection in the reinsurance market, the form of the contract is less important to our analysis than the quantity of reinsurance. Thus, for the sake of simplicity, we assume that reinsurance is transacted on a proportional (quota share) basis.

[7]Extending the model presented here to allow for the endogenous determination of the insurer's reinsurance, direct underwriting, investment, and capitalization choices is a potentially fruitful area for future research. However, we do not expect that the results of such an analysis would necessarily *qualitatively* invalidate the results obtained here.

equity of a levered firm represents a call option on the terminal value of the firm, with an exercise price equal to the face value of debt. Galai and Masulis (1976) combine Merton's (1973) continuous time CAPM with the Black-Scholes option pricing model in order to value levered equity and investigate the valuation and risk effects of changes in corporate investment policy. A number of authors (e.g., Galai (1983), Green and Talmor (1985), MacMinn (1987), Majd and Myers (1985), Pitts and Franks (1984), and Smith and Stulz (1985)) subsequently extended the contingent claim formulation of the firm's capital structure to a valuation of the government's tax claim. Our approach here is quite similar. Specifically, we view shareholders as holding a long position in a call option on the pre-tax terminal value of firm and a short position in its taxable income. However, the solution to our problem differs from previous approaches because the payoffs on the call options modeled here depend upon the outcomes of <u>two</u> random variables (investment returns and claims costs) rather than just one. Analytically, this implies that these call options have stochastic exercise prices.[8] In the valuation problem at hand, there is one exercise price common to both options, the value of which is determined in part by the realization of claims costs that have not been reinsured.

Next, the assumptions and notation that will be needed are presented.

2.1. Model Assumptions

The model assumptions are fivefold:

1. Single period;
2. Competitively structured financial markets and insurance markets;
3. Insurers are subject to the risk of insolvency;
4. Reinsurers are not subject to the risk of insolvency;

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[8]Fischer (1978) was the first researcher to address the pricing of an option with a stochastic exercise price. However, because his work is a direct extension of Black and Scholes, it relies primarily upon the stochastic calculus. Stapleton and Subrahmanyam (1984, pp. 223-224) present an alternative solution using preference-restricted contingent claims models based upon multivariate normal and lognormal density functions. See Doherty and Garven (1986) for an application of the Stapleton and Subrahmanyam framework to the pricing of property-liability insurance.

5. Investors' utility functions exhibit constant absolute risk aversion and investment returns, claims costs, and terminal wealths are multivariate normally distributed.

Some explanation of the model assumptions is warranted. Assumptions 2 and 5 are required in order to justify the use of the preference-restricted contingent claims model.[9] Assumptions 3 and 4 are made for the sake of analytic simplicity. Essentially, if reinsurance is risky, then the ceding insurer's policies become compound rather than simple options because their payoffs are functionally related to the default risk of both the ceding insurer and its reinsurer.

2.2. Model Notation

The following notation will also be needed:

S_0 = initial (exogenously determined) equity capital, or surplus of the insurer;

a = the proportion of the firm's liabilities that are to be reinsured, $a \in [0, 1]$ (the firm's decision variable);

$p(a)$ = default cost function, $p'(a)<0$;[10]

$P(a) = P - p(a)$ = gross premium income, $P'(a) > 0$;[11]

P_r = the (exogenously determined) price of a reinsurance contract that completely insures the firm's liabilities, paid at t=0;

$P_n(a) = P(a) - a\ P_r$ = net premiums written, $P_n(a) < 0$;[12]

$A(a) = S_0 + kP_n(a)$ = the insurer's beginning-of-period assets, $A'(a) < 0$;

[9]Our model is based upon the preference-restricted contingent claims model, developed for the univariate case by Brennan (1979) and extended to the multivariate case by Stapleton and Subrahmanyam (1984). Thus, we assume that investors have constant absolute risk aversion (CARA) preferences, and that future random cash flows, consisting of investment returns and claims costs, are normally distributed. Brennan (1979) notes that the discrete time model is particularly appropriate when valuing non-traded claims such as the firm's taxable income. Also see Doherty and Garven (1986) for their application of this framework to the "fair return" problem in property-liability insurance.

[10]Default-related costs include costs related to agency problems as well as explicit transaction costs that are incurred when firms suffer bankruptcy, such as lawyers' and accountants' fees, other professional fees, court costs, and the value of the managerial time spent in administering the bankruptcy.

[11]In the expression $P(a) = P - p(a)$, P is the (exogenously determined) gross premium income which would obtain in a default-free setting in which all tax shields are fully utilized. Its value is therefore equal to $E(L)/(1\text{-}E(r_u))$, where $E(r_u) = -kr_f(1-qt)/(1-t) + b_u[E(r_m) - r_f] + (V_e/P_0)r_f(qt/(1-t))$ (see Garven (1987), section 2.2. for the development of this particular formula). $P'(a) > 0$ because higher values of make it possible for the firm to charge higher premiums.

[12]Although the purchase of reinsurance lowers various costs related to agency and tax effects, it also decreases net premiums written and consequently the total amount of capital that can be invested in the financial market. Note that assumptions 2 and 3 imply that $P(a{=}1) = P_r$; consequently, if the firm complete reinsures, then $P_n(a) = 0$.

k = the funds generating coefficient, or average claim delay;[13]
q = the proportion of the firm's investment income that is subject to taxation; $q \in [0,1]$;

$f(r_p,L)$ = bivariate normal density function governing the insurer's investment returns (r_p) and claims costs (L);

$\hat{f}(r_p, L)$ = the corresponding risk-neutral bivariate density function;[14]

r_f = the riskless rate of interest;

r_m = the rate of return on the market portfolio;

$R_i = 1 + r_i$, $i = f,m,p$;

$n(\bullet)$ = standard normal density function;

$N(\bullet)$ = cumulative standard normal distribution function.

2.3. The Value of the Insurance Firm

The value of the pre-tax equity claim will be referred to as $C(AR_p;-U)$, where $AR_p = A(1+r_p)$ is the pre-tax terminal value of the insurer's investment portfolio (i.e., the value of the underlying asset), and $-U = (1-a)L - P_n$ is the negative of the insurer's pre-tax underwriting income (i.e., the exercise price). The value of the government's claim will be subsequently referred to as $tC(Aqr_p;-U)$, where Aqr_p is the terminal value of taxable investment income.

More formally, the pre-tax value of equity, $C(AR_p;-U)$, can be written as follows:

[13] Depending upon the type of risk being insured, the time lag between the receipt of the premium and payment of the claim can vary considerably. For example, most casualty insurance lines are characterized by claim delays of less than one year, whereas most liability lines have claim delays of more than one year. Consequently, for every dollar of premiums written, lines of insurance with longer claim delays generate more investable funds than insurance lines with shorter claim delays. Therefore, the "funds generating coefficient" can be interpreted as the average amount of investable funds per dollar of annual premiums. This type of adjustment is common throughout most of the financial pricing literature (e.g., see Doherty and Garven (1986)).

[14] A "risk neutral" density function is a density function whose location parameter is chosen so that the mean of the distribution is its certainty equivalent (see Brennan (1979) and Stapleton and Subrahmanyam (1984)). In the case of a multivariate risk neutral density function, the same result holds for the means of the marginal density functions.

$$C(AR_p;-U)=R_f^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} MAX\left[\left(AR_p+P_n-(1-a)L\right),0\right]\hat{f}(r_p,L)dr_p dL. \tag{1}$$

From equation (1), it is apparent that if the terminal value of cash flow derived from the firm's investment, underwriting, and reinsurance activities is non-negative, then shareholders will have a valuable claim. However, if cash flow fails to assume a positive value, then shareholders will exercise their "limited-liability option" by declaring bankruptcy.

Since R_p and L are normal variates, it is convenient to solve equation (1) by defining a normal variate $Y = AR_p - (1-a)L$, with certainty-equivalent expectation $\hat{E}(Y)=AR_f-(1-a)\hat{E}(L)$, variance $s_Y^2 = A^2 s_p^2+(1-a)^2 s_L^2 -2A(1-a)s_{pL}$, and risk-neutral density $\hat{f}(Y)$. Then equation (1) reduces to

$$C(AR_p;-U)=R_f^{-1}\int_{-P_n}^{\infty}(Y+P_n)\hat{f}(Y)dY. \tag{2}$$

Changing the random variate Y to a standardized normal variate y and solving yields

$$C(AR_p;-U)=\left\{A+\left[P_n-(1-a)\hat{E}(L)\right]R_f^{-1}\right\}N(X_1)+R_f^{-1}s_y n(X_1), \tag{3}$$

where

$X_1 = \left[AR_f+P_n-(1-a)\hat{E}(L)\right]/s_y$ = the standardized certainty-equivalent terminal value of pre-tax profit;

$N(X_1)$ = the pre-tax certainty-equivalent terminal value of one dollar invested in the firm, provided the firm remains solvent.[15]

Similarly, the value of the government's claim, $tC(Aqr_p;-U)$, can be written as:

[15] $N(X_1)$ is not the probability of solvency; rather, the probability of solvency is given by $N(X_1^*)$, where $X_1^* = [AE(R_p) + P_n - (1-a)E(L)]/s_y$. Because $N(X_1)$ is in effect a "risk neutral" cumulative distribution function, it understates the solvency probability by the amount of risk bearing costs borne per dollar of income generated in solvent states of nature; i.e., by the difference $N(X_1^*) - N(X_1)$.

$$t\ C(Aqr_p;-U) = t\ R_f^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} MAX\left[\left(Aqr_p + P_n - (1-a)L\right),0\right]\hat{f}(r_p, L)dr_p dL. \qquad (4)$$

From equation (4), it is readily apparent that if the terminal value of taxable income is non-negative, then the government will have a valuable claim. However, if taxable income fails to assume a positive value, then shareholders will exercise their "tax exemption option". Thus the model allows for the existence of a state interval in which tax-shields (e.g., investment and/or underwriting losses) are underutilized.

The solution of equation (4) requires that a normal variate Z be defined, where Z = Aqr_p - $(1-a)L$. The certainty-equivalent expectation of Z is $\hat{E}(Z) = Aqr_f - (1-a)\hat{E}(L)$, its variance is $s_z^2 = A^2q^2s_p^2 + (1-a)^2 s_L^2 - 2Aq(1-a)s_{pL}$, and its risk-neutral density is $\hat{f}(Z)$. Substituting this change of variables into (4) yields (5):

$$t\ C(Aqr_p;-U) = t\ R_f^{-1}\int_{-P_n}^{\infty}(Z+P_n)\,\hat{f}(Z)dZ. \qquad (5)$$

Changing the random variable Z to a standardized normal variate z and solving yields

$$t\ C(Aqr_p;-U) = t\ R_f^{-1}\left\{Aqr_f + P_n - (1-a)\hat{E}(L)\right\}N(X_2) + t\ R_f^{-1}s_z n(X_2), \qquad (6)$$

where

$X_2 = \left[Aqr_f + P_n - (1-a)\hat{E}(L)\right]/s_z$ = the standardized certainty-equivalent terminal value of taxable profit;

$N(X_2)$ = the certainty-equivalent terminal value of one dollar of taxable profit, provided that tax-shields are fully utilized.[16]

[16] $N(X_2)$ has a similar interpretation to the interpretation given $N(X_2)$. That is, $N(X_2)$ has a "risk averse" counterpart $N(X_2^*)$ which represents the probability of taxation, where $X_2^* = [AqE(r_p) + P_n - (1-a)\mathrm{E}(\mathrm{L})]/s_z$. $N(X_2)$ understates the probability of taxation by the amount of risk bearing costs borne per dollar of taxable income generated in taxable states of nature; i.e., by the difference $N(X_2^*) - N(X_2)$.

Since shareholders hold a long position in $C(AR_p;-U)$ and a short position in $tC(Aqr_p;-U)$, the after-tax value of equity, V_e, is found by subtracting $tC(Aqr_p;-U)$ from $C(AR_p;-U)$:

$$\begin{aligned} V_e = &\left\{ A + \left[P_n - (1-a)\hat{E}(L) \right] R_f^{-1} \right\} N(X_1) + R_f^{-1} s_y n(X_1) \\ &- t\, R_f^{-1} \left\{ Aqr_f + P_n - (1-a)\hat{E}(L) \right\} N(X_2) + t\, R_f^{-1} s_z n(X_2). \end{aligned} \tag{7}$$

The firm's optimal reinsurance decision maximizes V_e. To see how the purchase of reinsurance affects V_e, the first-order condition is calculated by differentiating equation V_e in equation (7) with respect to a:

$$\begin{aligned} \frac{\P V_e}{\P a} = &-P_r \left[N(X_1) - tR_f^{-1} N(X_2) \right] + \left[\hat{E}(L) R_f^{-1} - \frac{\P p}{\P a} \right] \left[N(X_1) + tR_f^{-1} N(X_2) \right] \\ &- k \left[\frac{\P p}{\P a} + P_r \right] \left[N(X_1) - qtr_f R_f^{-1} N(X_2) \right] + R_f^{-1} \left[\frac{\P s_y}{\P a} n(X_1) - t\frac{\P s_z}{\P a} n(X_2) \right]. \end{aligned} \tag{8}$$

The optimal value of a exists at the point at which the firm's after-tax value of equity decreases with any further change in its reinsurance coverage. This occurs at the point at which the expected after-tax marginal costs of reinsurance and risk retention are equal.[17]

The four bracketed terms in equation (8) can interpreted in the following manner:

1. Holding the probabilities of solvency and taxation constant, the first term represents the after-tax marginal cost of reinsurance.

2. Holding the probabilities of solvency and taxation constant, the second term represents the after-tax marginal benefits of lower claims and agency costs obtained from reinsuring.

3. Holding the probabilities of solvency and taxation constant, the third term represents the after-tax marginal cost associated with foregone investment income. Investment income is foregone

[17]This statement assumes, of course, that the second order condition for a maximum is also satisfied. Analytically, the second order condition is not trivially satisfied; however, it obtains for most reasonable parameterizations of the model.

for the simple reason that the purchase of reinsurance reduces the amount of money that can be invested in the financial market.

4. Holding the expected values of pre-tax profit and taxable income constant, the fourth term represents the effects that changes in the variability of pre-tax income and taxable income have upon the value of the firm. This term essentially provides an indication of how much more or less valuable the limited-liability and tax exemption options become as more reinsurance is purchased. As long as investment returns and claims costs are negatively correlated, then the purchase of reinsurance decreases the value of the limited-liability option and increases the value of the tax exemption option. However, if positive correlation exists between r_p and L, then the opposite will occur.

3. TESTABLE HYPOTHESES

In the previous section, we showed that the optimal reinsurance decision of a limited-liability insurer is influenced by factors such as its investment-return and claims-cost distributions, the magnitude of tax-shields derived from its investment and underwriting activities, agency costs, and default risk. The purpose in this section of the paper will be to briefly outline various testable hypotheses that are yielded by the comparative statics of the model.

From equation (8), the following set of cross-sectional predictions is derived:[18]

<u>HYPOTHESIS 1:</u> Other things equal, the demand for reinsurance will be greater the higher the firm's leverage;

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[18]Hypotheses 1-4 were derived from a comparative static analysis of the first order condition provided in equation (8). Since no closed form solution exists for the optimal reinsurance decision, the comparative static analysis was accomplished by using the implicit function theorem. The implicit function theorem states that given some function $F(y,x_1,\ldots, x_n) = 0$, if an implicit function $y = f(x_1,\ldots, x_n)$ exists, then the partial derivatives of the implicit function are:

$$\frac{\partial y}{\partial x_i} = -\frac{\partial F / \partial x_i}{\partial F / \partial y},$$

for all i, $i = 1,\ldots,m$. In the model described above, the first order condition is $V_\alpha^e(\alpha^*, x_1,\ldots,x_m)=0$, where V_α^e corresponds to the partial derivative of equity value with respect to α, the x_i's represent model parameters, and $\alpha^* = f(x_1,\ldots,x_m)$ is the implicit function. Therefore,

$$\frac{\partial \alpha^*}{\partial x_i} = -\frac{\partial V^e / \partial x_i}{\partial V^e / \partial \alpha}$$

for all i, $i = 1,\ldots,m$. Since $\partial^2 V^e / \partial \alpha^2 < 0$ when evaluated at α^*, this means that the sign of $\partial \alpha^* / \partial x_i$ will be the same as the sign of $\partial^2 V^e / \partial \alpha \partial x_i$. A mathematical appendix which provides detailed proofs of these hypotheses is available upon request.

<u>HYPOTHESIS 2:</u> Other things equal, the demand for reinsurance will be greater the lower the correlation between the firm's investment returns and claims costs.

<u>HYPOTHESIS 3:</u> Other things equal, the demand for reinsurance will be greater for firms that write "longer-tail" lines of insurance;

<u>HYPOTHESIS 4:</u> Other things equal, the demand for reinsurance will be greater for firms that concentrate their investments in tax-favored assets.

Hypothesis 1 highlights the fact that reinsurance is essentially a substitute for surplus in terms of its leverage effect; i.e., the lower the level of surplus, the higher the firm's financial leverage. Higher leverage results in a lowering of the probability of solvency and an increase in the probability of tax-shield underutilization, which in turn results in an increase in the demand for reinsurance.

The intuition behind Hypothesis 2 is also quite appealing. With negative correlation, the values of both the pre-tax equity claim and the government's claim will fall due to the fact that the variances of pre-tax income and taxable income will also decline. This is analogous to the well-known comparative-static relationship between the value of a call and the variance rate of the underlying asset. However, with positive correlation, the firm is provided with a natural hedge if it retains risk. By reinsuring, the natural hedge may be destroyed, thereby increasing the variances of pre-tax income and taxable income and hence the values of both the pre-tax equity claim and the government's claim.

Hypothesis 3 can also be explained in terms of a leverage effect. Although an increase in the average claim delay lowers the firm's premium income and reinsurance premiums, it also causes more investable funds to be generated per dollar of premiums. Overall, this latter effect dominates the former, leading to a net increase in financial leverage. Hence one would expect to observe a greater propensity toward reinsurance purchases by firms that underwrite risks with longer claim delays. Specifically, firms that specialize in liability risks should purchase more reinsurance than should firms specializing in property risks.[19]

The rationale for Hypothesis 4 can be best explained by first invoking a basic principle of asset pricing; that is, in equilibrium, after-tax certainty-equivalent returns must be equal across all securities. In the case of two financial assets that differ only with respect to taxation, the expected yield on the more fully taxed instrument should be grossed up to completely offset its marginally higher tax burden. Therefore, although the firm may gain valuable tax-shields by lowering q, such an action simultaneously decreases the probabilities of solvency and taxation due to the commensurately lower investment-return prospects. Hence, as in the case of an increase in the average claim delay, a decrease in the "tax-shield" coefficient should result in an increase in the demand for reinsurance.

Previous empirical research by Mayers and Smith (1990) documents that factors such as ownership structure, firm size, geographic concentration, and line-of-business concentration also influence the demand for reinsurance.[20] Since these variables are known to be important cross-sectionally, the empirical study that follows will use these factors as control variables. Mayers and Smith note that their analysis is limited by the fact that their data do not allow them to distinguish between firms on the basis of 1) tax status, 2) cash flow volatilities, and 3) within-line policy heterogeneity.[21] The present study seeks to address all but the third limitation. Specifically, hypothesis 4 addresses the first limitation, since the decision to purchase tax-favored assets is obviously significantly influenced by the firm's tax status. Hypothesis 2 addresses the second

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[19]In the empirical tests that follow, this effect will be captured by a variable that measures the proportion of the firm's total premiums accounted for by premiums written in Schedule P lines.

[20]Mayers and Smith also find that reinsurance demand is negatively related to Best's Ratings. Since firms with low Best's Ratings will, on average, tend to be more highly leveraged than firms with high Best's Ratings, the Best's Rating factor essentially proxies for the effect predicted in Hypothesis 1.

[21]Due to different policy forms within a line of business, diversification may occur. This level of detail is not available in most publicly available in most insurance databases, including ours.

limitation, since the calculation of the correlation between investment returns and claims costs involves the estimation of cash flow volatilities.

Mayers and Smith also note that since their data do not classify interfirm reinsurance transactions differently from intrafirm transactions, it is not possible to directly compare the reinsurance behavior of unaffiliated single companies and insurance groups. In view of this fact, we have omitted groups and only consider unaffiliated single companies.

4. DATA AND EMPIRICAL RESULTS

4.1. Panel Data

Eight years of data were obtained from A. M. Best Company's 1980-1987 Balance Sheet-Income Statement and Premium-Losses-Expenses databases. The criteria applied in the selection of the sample were as follows:

1. The firm must be an unaffiliated single company.
2. The firm must have been classified as either a stock or mutual company during the entire eight-year period. Furthermore, it must not be classified as a specialist reinsurer.
3. Since a number of variables in the regression model involve ratios, only those firms reporting positive (nonzero) values for the denominators of these ratios are included in the sample so as to avoid division by zero.

The application of these criteria resulted in a sample of 179 insurers. Summary statistics for these firms are presented in Table 1.

4.2. Panel Model

The following regression model was estimated for the sample consisting of panel data:

$$REINS_j = b_{0j} + \sum_{i=1}^{17} b_{ij} X_{ij} + e_{ij}, \tag{9}$$

where

$REINS_j$ = ratio of reinsurance premiums ceded to total business premiums for firm j;
X_{1j} = $SIZE_j$ = natural logarithm of firm j's size, measured in terms of admitted assets;
X_{2j} = $PSRATIO_j$ = ratio of direct premiums written to surplus for firm j;
X_{3j} = RHO_j = correlation between investment returns and claims for firm j;[22]
X_{4j} = $STDP_j$ = standard deviation of firm j's investment returns;
X_{5j} = $STDL_j$ = standard deviation of firm j's claims costs;
X_{6j} = $SCHEDP_j$ = proportion of firm j's premiums written in Schedule P lines;
X_{7j} = $THETA_j$ = firm j's tax-shield coefficient q, which measures the proportion of investment income subject to taxation;[23]

$$X_{8j} = HERF_j = \text{firm j's Herfindahl index} = \sum_{i=1}^{n}\left(\frac{(\text{Direct Premiums Written})_{ij}}{\sum_{i=1}^{n}(\text{Direct Premiums Written})_{ij}}\right)^2;$$

X_{9j} = $LICENSE_j$ = the negative of the number of states in which firm j is licensed;
X_{10j} = $MUTUAL_j$ = 1 if firm j is a mutual, 0 if firm j is a stock insurer;
X_{11j} -X_{17j} = T_1- T_7 = year indicators; T_1=1 if $YEAR$ = 1981, 0 otherwise, ..., T_7=1 if $YEAR$=1987, 0 otherwise.

The variables $REINS_j$, $SIZE_j$, $HERF_j$, $LICENSE_j$ and $MUTUAL_j$ were measured in the same manner as in the Mayers and Smith (1990) study. Since the model presented in the previous section assumes the purchase of proportional reinsurance, the Mayers and Smith definition for $REINS_j$ most closely fits the theory; viz., $REINS_j$ = 1 if a_j = 1 and $REINS_j$ = 0 if a_j = 0. $SIZE_j$,

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[22] RHO_j was measured in the following manner:

$$r_{PL}^{j} = RHO_j = \sum_{\substack{i=1 \\ i\neq k}}^{14}\sum_{k=1}^{17} X_{ij}W_{kj}COV(r_{ij}, L_{kj}) / s_{pj}s_{L_j};$$

where

X_{ij} = proportion of firm j's assets invested in asset class i;
r_{ij} = the return on firm j's ith investment;
r_{pj} = the return on firm j's portfolio of investments = $\Sigma X_{ij}r_{ij}$;
s_{pj} = standard deviation of r_{pj};
W_{kj} = proportion of firm j's written premiums due to line k;
L_j = firm j's claims costs = $\Sigma W_{kj}L_{kj}$;
s_{Lj} = standard deviation of L_j.

The data for these calculations were obtained from a variety of sources. The liability covariances were based upon aggregate industry data for 1970-1994, whereas asset covariances were based upon 300 monthly observations per asset type during this same period (source: Ibbotson and Associates (2002)). Asset/liability covariances were calculated from 25 annual observations obtained from aggregate industry data and Ibbotson and Associates. Firm-specific proxies for asset variances, liability variances, and asset-liability covariances based upon applying the formula above using X and W values from the A. M. Best Balance Sheet and Income Statement database.

[23] $THETA_j$ was determined by dividing taxable investment income into total investment income. See D'Arcy and Garven's (1990) appendix for further details on the mechanics of this particular calculation.

$HERF_j$, $LICENSE_j$ and $MUTUAL_j$ are controls for cross-sectional variation in size, line-of-business concentration, geographic concentration, and ownership structure. The remaining set of control variables is T_1-T_7, which control for time. When T_1-T_7 and $MUTUAL_j$ are all turned off, this implies that the data for a stock insurer is being observed for the year 1980. Consequently, the coefficients associated with these variables measure the extent to which the mean value of $REINS$ differs over time and between stock and mutual organizations, whereas the associated t statistics test whether these differences are statistically significant.

A second regression was estimated for the sample consisting of panel data whereby the standard deviation of equity, $STDE_i$, serves a proxy for cash flow volatility.[24]

4.3. Empirical Results

Table 2 provides the regression parameter estimates, standard errors, t statistics, and two-tail probabilities for the panel experiments.

The regression equation obtained from the first panel experiment has an adjusted R^2 value of .3207 and the F statistic is statistically significant at the .0001 level. The coefficients associated with the control variables suggested by Mayers and Smith are generally consistent with the findings of their study; specifically, size, line-of-business concentration and geographic concentration have a significant negative impact on the demand for reinsurance. Although the panel experiment failed to find any evidence in support of a difference in the demand for reinsurance between stock and mutual insurers, it should be noted that the differences observed by Mayers and Smith result from their use of a more detailed stock-ownership metric that classifies stock insurers into four subcategories: widely held, closely held, single-owner, and association-owned stocks. Also the time controls are generally insignificant, which suggests that the cross-sectional relations are stable

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[24] $STDE_j$ was calculated according to the formula $STDE_j = \sqrt{s_P^2 + s_L^2 - 2 s_P s_L r_{PL}}$.

over time. The only exception in this regard is the time control for 1987 (T_7), which is most likely due to the structural change in the tax code brought about by the Tax Reform Act of 1986.[25] As a result of this tax act, reinsurance became marginally more valuable from a hedging perspective.

Turning to the model predictions, we find that, as predicted, leverage (*PSRATIO*), asset volatilities (*STDP*) and length of tail (*SCHEDP*) have a significant positive impact on the demand for reinsurance. Although the parameter estimate associated with the tax variable *THETA* is of the correct sign, it does not differ significantly from zero. The parameter estimate associated with *RHO* has a significant negative impact on the demand for reinsurance. That is, the higher the covariance between assets and liabilities, the greater the natural hedge; consequently, the lower the demand for reinsurance.

Next, consider the results obtained from the second panel experiment whereby the standard deviation of equity proxies cash flow volatilities. The regression equation has an adjusted R^2 value of .309 and the F statistic is statistically significant at the .0001 level. Although this experiment yields similar results concerning the effects of size, geographic concentration, leverage, and length of tail as in the first panel experiment, the parameter estimate associated with line of business concentration (*HERF*) is not significantly different from zero. As predicted, equity risk (*STDE*) has a significant positive impact on the demand for reinsurance. One meaningful difference between the first and second experiment, is that when equity risk serves as the proxy for cash flow volatilities, ownership structure (*MUTUAL*) has a negative and significant effect on the demand for reinsurance. These results are consistent with the risk pooling hypothesis of Doherty and Dionne (1993).

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[25]Walker (1991) notes that the Tax Reform Act of 1986 appeared to be specifically targeted to significantly increase the burden of the corporate income tax on the U.S. property-liability industry in particular. Interestingly, during the 25 year period preceding the passage of this act, the net cumulative federal income tax liability of the industry was approximately zero, a fact that was probably not lost on Congress when the law was enacted.

5. SUMMARY AND CONCLUSION

The purpose of this paper has been to provide a theoretical and empirical analysis of the demand for reinsurance. The comparative statics of the theoretical model suggested that factors such as leverage, the correlation between investment returns and claims costs, length of tail, and tax status influence the demand for reinsurance. Although the empirical analysis focused on these factors, controls were also implemented for other factors that have also been shown to be important, such as ownership structure, firm size, geographic concentration, and line-of-concentration.

The empirical evidence strongly supports most of the model's predictions. Although the tax-shield effect predicted by the model is not empirically supported, it appears that a tax effect related to the passage of the Tax Reform Act of 1986 exists which is not necessarily inconsistent with the tax effect hypothesized by the model. Furthermore, the rest of our hypotheses receive strong empirical support. Specifically, we find that the demand for reinsurance is positively or directly related to leverage, asset volatility, and length of tail (i.e., claim delay). We also find that the demand for reinsurance is negatively or inversely related to the correlation between investment returns and claims costs; i.e., as correlation increases, the demand for reinsurance decreases, other factors held constant.

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TABLE 1

Panel Data Summary Statistics

Variable	N	Mean	Standard Deviation	Minimum	Maximum
REINS	1350	0.2724	0.2163	-0.0912	1.0012
SIZE	1350	16.7714	1.3024	13.5140	20.7231
PSRATIO	1350	2.4754	1.6136	0.0078	9.9674
HERF	1350	0.4320	0.2164	0.0017	0.9988
RHO	1350	0.1145	0.1268	-0.3729	0.4017
STD_E	1350	0.1519	0.0926	0.0289	0.4925
STD_P	1350	0.0587	0.0254	0.0230	0.1690
STD_L	1350	0.0139	0.0219	0.0000	0.1206
SCHED_P	1350	0.6434	0.2414	0.0000	0.9980
THETA	1350	0.6983	0.2013	0.0303	1.0000
LICENSE	1350	-46.8393	13.9387	-56.0000	-2.0000
MUTUAL	1350	0.5970	0.4907	0.0000	1.0000

TABLE 2

Regression Parameter Estimates (Base Case)

$R^2 = 0.3207$

Panel Data	Parameter Est.	Standard Error	*t* Statistic	Prob > |*t*|
INTERCEPT	1.7587	0.1046	16.817	0.0001
SIZE	-0.0777	0.0050	-15.417	0.0001
PSRATIO	0.0239	0.0032	7.386	0.0001
RHO	-0.1077	0.0461	-2.333	0.0198
STDP	0.8948	0.2333	3.835	0.0001
STDL	0.3177	0.3232	0.983	0.3257
SCHEDP	0.0986	0.0285	3.458	0.0006
THETA	-0.0303	0.0261	-1.162	0.2456
HERF	-0.0460	0.0240	-1.916	0.0556
LICENSE	-0.0069	0.0004	-16.434	0.0001
MUTUAL	-0.0049	0.0112	-0.436	0.6630
T1	0.0056	0.0194	0.291	0.7710
T2	0.0138	0.0192	0.721	0.4711
T3	0.0330	0.0193	1.709	0.0876
T4	0.0101	0.0195	0.520	0.6031
T5	0.0147	0.0197	0.748	0.4545
T6	0.0321	0.0198	1.624	0.1046
T7	0.0419	0.0198	2.109	0.0351

TABLE 2 (CONTINUED)
Regression Parameter Estimates (Equity Risk)
$R^2 = 0.3097$

Panel Data	Parameter Est.	Standard Error	*t* Statistic	Prob > |*t*|
INTERCEPT	1.8166	0.1060	17.123	0.0001
SIZE	-0.0828	0.0049	-16.740	0.0001
PSRATIO	0.0273	0.0032	8.412	0.0001
STDE	0.2278	0.0751	3.033	0.0025
SCHEDP	0.1408	0.0274	5.138	0.0001
THETA	-0.0285	0.0255	-1.117	0.2640
HERF	-0.0379	0.0239	-1.585	0.1132
LICENSE	-0.0068	0.0004	-16.165	0.0001
MUTUAL	-0.0201	0.0110	-1.820	0.0690
T1	0.0065	0.0195	0.334	0.7386
T2	0.0161	0.0193	0.836	0.4035
T3	0.0305	0.0194	1.568	0.1172
T4	0.0121	0.0196	0.618	0.5369
T5	0.0185	0.0198	0.938	0.3482
T6	0.0381	0.0199	1.914	0.0558
T7	0.0466	0.0199	2.335	0.0197