Agree to Disagree: Heterogeneous Beliefs, Stochastic Liabilities, and Shareholder Wealth

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October 19, 2022

Abstract

Firms with stochastic liabilities can enhance shareholder wealth by trading with counterparties who place different values on such liabilities because of heterogeneous beliefs or idiosyncratic risk management opportunities. We illustrate this by showing an exchange transaction under which the issuance of a catastrophe bond (which increases outstanding equity holdings) may have such an effect when used to shift risky liabilities to those new synthetic equity holders. Under heterogeneous beliefs, there may be other swap-like transactions in which exchanges in levels of stochastic liabilities may either enhance or diminish shareholder wealth.

1 Introduction

Corporate bonds typically feature fixed payments and face values. While bonds may also be callable and have adjustable interest rates, for most investors, the primary source of firm-specific risk is the risk of default. On the other hand, many corporate liabilities are stochastic in the sense that the amount and timing of payments may also be contingent on factors other than default risk. Insurance reserves and bank demand deposits represent examples of corporate obligations for which terminal payments and maturities vary based on various other underlying factors. While defined benefit plans have largely fallen out of favor, some employers continue to hold pension obligations on their balance sheets. Pension plans feature similar stochastic characteristics, primarily related to longevity risk of the pension

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participants, as well as volatility of contractual cost-of-living adjustments that may be linked to an index such as the Consumer Price Index. Managers can either retain such liabilities or transfer them contractually or via securitization. This study provides a general model for evaluating the impact of such decisions under heterogeneous beliefs.

An insurer's policyholders can only make claims when they experience insured losses, which vary in both value and time. Similarly, banks must satisfy claims on demand deposits at the discretion of depositors, and firms with pension obligations do not know *a priori* how long pensioners will live to collect on their pensions and sometimes subject to variable cost of living adjustments. Regulators and ratings agencies monitor insurers, banks, and pension plans to ensure solvency in the face of large contingent claims; properly implemented, regulatory oversight increases the likelihood that claims will be honored when they arise. Various securitization methods (catastrophe bonds, collateralization, pension conversions and terminations, etc.) enable firms to dynamically manage asset and liability risks. However, such moves typically involve cash transfers, which can be implemented by using slack cash or by issuing debt or equity.

Transactions such as these represent forms of debt-equity or equity-debt swaps; i.e., the exchange of debt for equity or equity for debt.[1] For example, insurers can exchange debt for equity by expanding net premiums written and using underwriting profits to retire equity. Similarly, by warehousing rather than securitizing assets such as mortgages, auto loans, and credit card receivables, banks effectively exchange debt for equity, as pension plans could (in theory) by expanding their defined benefit pension plan offerings. Managers may undertake exchanges when debt ratios deviate from the firm's target capital structure. While signaling theory suggests that opportunities may be limited, exchanges may also be appropriate when managers observe security prices that are not in line with their own beliefs about potential future states.[2]

[1]For simplicity, we will refer to any transaction that shifts assets to claims (liabilities and equity) or among claims as "exchanges".

[2]We note that even if managers use slack cash to facilitate such a transaction, it is still an exchange in that the cash could have also been used to pay dividends or repurchase shares.

In this article, we examine the impact of heterogeneous beliefs and/or idiosyncratic opportunity (referred to here as differential risk-bearing capacity or differential risk preferences for managers or shareholders) on potential benefits associated with balance sheet exchanges. Our definition of heterogeneous beliefs closely follows the definition of differences of opinion provided by Harris and Raviv (1993): "We assume that traders share common prior beliefs and receive common information but differ in the way in which they interpret this information". We develop a model that managers can use to analyze changes in their estimates of shareholder wealth resulting from increasing or decreasing stochastic liabilities. Prior literature regarding exchanges focused on information asymmetry (primarily around asset values and expected cash flows from projects), however, in our model, the primary *source* of value arises from gains from trade based on heterogeneous beliefs or differential predicted portfolio impacts between insiders and outsiders. Specifically, we show that if outsiders' beliefs differ from managers about joint probability distributions for the impact of the firm's stochastic liabilities on their own balance sheets, the wealth of long-term shareholders can be positively affected by dynamically retiring or issuing equity as appropriate.

In this article, we motivate the idea of exchanges within the broader context of capital structure theory through an example of catastrophe securitization or catastrophe bond issue. While catastrophe bonds vary significantly in perils covered, rates of return, triggers, and other features, we use a simple model of a single-tranche, single-peril, parametric trigger catastrophe bond.[3] We also suggest some transactions that could raise cash backed by risky liabilities when the newly raised cash is used to repurchase shares. Further, we develop the theoretical structure of the model, and present numerical illustrations of various exchange scenarios. The model has practical implications for managers evaluating conversions of stochastic liabilities to fixed liabilities or equity capital. Under certain scenarios, we obtain the counter-intuitive result that issuing *undervalued* equity may increase long-term shareholder wealth.

[3]For an accessible discussion of catastrophe bond features and problems, see Lakdawalla and Zanjani (2012).

2 Motivation

The dramatic growth of the catastrophe bond market since the Great Recession of 2007-2009, the assumed role of collateralized debt obligations (CDOs) in the bank failures that precipitated the financial crisis, and the high rate at which firms replaced their traditional defined benefit plans with cash balance or defined contribution plans (Harper and Treanor (2014)) inform the goals of this study. We propose a model by which managers can evaluate the impact of adjusting the balance of risky obligations within their overall risk profile. In a real options approach to valuing a firm, the relationship between asset and liability risk is especially important, and our models highlight their relative sensitivities.

Balance sheet exchange analysis is often clouded by the presence of information asymmetry (which can generate problems with moral hazard, adverse selection, and inferences about information drawn from signalling behavior). By using a simple catastrophe bond for modeling purposes, we are able to abstract away from these complications. Rather, we consider the role of potential *heterogeneous beliefs* (which may result from managers employing different catastrophe models, for example) or *idiosyncratic opportunity* (resulting from low correlation with the counterparties' asset portfolio or the counterparties' risk preferences or risk-bearing capacity). The simple catastrophe bond setting that we use features no default risk, no moral hazard, few (if any) signalling issues, and a simple transfer of risk.

Additionally, we contribute to the scholarly discussion of how managers can enrich shareholders through dilutive equity issues. While there is substantial evidence that share prices tend to fall upon announcement of an equity issue, we focus on the terminal value of the firm, when all claims are settled and the effect of the transaction set is fully known. We demonstrate how a real option approach to firm valuation allows risk contributions to be incorporated into the value when choosing to adjust capital structure in certain scenarios.

The analysis is further motivated by the observation of Babbel and Merrill (2005), who show that the market value of a firm's assets is the sum of its franchise value and the market value of its tangible assets, while its market value of liabilities is the expected present value of the firm's liabilities.[4] We model the firm's equity as an option to use its assets to settle stochastic liabilities, or transfer them to a counterparty prior to their due date. Thus, we extend the existing literature by examining the availability of liability securitization as a leverage-reducing tool for such firms.

Rather than focus on the franchise value, however, we model changes to the value of the exchange option *per se*. In our model, the implied value of liabilities is the difference between the value of the firm's assets and the value of the exchange option held by the obligee. Since a catastrophe bond allows an obligee to lay off risks in exchange for a premium payment, leverage reduction may be possible without public announcement (even though many catastrophe bonds are tracked and reported on by publicly available sources such as artemis.bm). Including the covariance between the firm's assets and liabilities, we examine situations in which the firm can benefit from issuing a catastrophe bond simply because of the investor's comparative advantage in bearing risk. We also examine the potential shareholder wealth effects from management trading on heterogeneous beliefs among the managers, the investors, and possibly even other stakeholders.

Prior work provides guidance for this study. Fischer (1978) develops an extension to the Black-Scholes (Black and Scholes, 1973) option pricing model, in which the risk-free rate is replaced by the difference between the expected return on a hedge asset and the expected rate of increase of the exercise price of the option. However, difficulty finding an appropriate hedge asset for stochastic liabilities limits the usefulness of the Fischer model for this application. Merton (1973) and Margrabe (1978) suggest an extension of the Black-Scholes option pricing model that estimates the value of an option to exchange one asset for another, when the value of each asset is a random variable. Margrabe specifically suggests that this model is appropriate for valuing margin accounts, which accurately describes any firm with risky financial or operating leverage. We will refer to this model as the "Merton-Margrabe" model.

[4] While Babbel and Merrill (2005) explicitly discuss insurers, their observations about franchise value can easily be extended to any type of firm.

Catastrophe bonds are often considered a substitute for insurance. While Froot and O'Connell (2008) and Weiss and Chung (2004) show that reinsurance (and by extension, insurance) is typically priced at levels above expected loss, Froot and O'Connell suggest that the transparency and standardization offered by catastrophe bonds may reduce the costs of intermediated risk transfer. For example, firms with concentrated ownership (e.g., privately held firms, hedge funds, Lloyd's syndicates, risk retention groups, etc.) and firms with less geographic concentration may have few diversification opportunities, making risk reduction appealing. Managers could add value for long-term shareholders through a risk transfer to an agent with comparative advantage in risk-bearing (one with greater diversification opportunities) or efficiency in providing real services (economies of scale allow a catastrophe bond sponsor to model risks at lower marginal cost). Throughout this article, we refer to such mutually beneficial transfers as exchanges with agents with idiosyncratic opportunities. The relationship between the ceding company's obligations and the assuming investor's asset/liability portfolio may allow that investor to take on these risks at a lower total cost than the ceding company's cost of carrying such risky obligations. This may be especially true for certain tranches of a catastrophe bonds in which the ceding company transfers only tail risk to the investor. Alternatively, the ceding company management may believe that the expected loss or volatility of the risky liability differ from those modeled by the catastrophe bond sponsor or the assuming investor. Any of these cases may create capital structure arbitrage opportunities similar to those suggested by O'Brien, Klein, and Hilliard (2007) for fixed liability obligations. We note that signaling equilibrium problems may still be present, but are mitigated to some extent by the fully collateralized nature of catastrophe bonds and heterogeneous beliefs held by counterparties.

3 Theoretical Development

Consider a firm with liquid assets invested in a "market portfolio" of tradable securities and stochastic liabilities whose value will depend on whether a hurricane hits the Gulf Coast with landfall. The firm has other liquid and fixed assets as well as non-contingent liabilities. The market portfolio and contingent liability each contribute correlated risk to the firm's balance sheet.

Management's objective is to maximize the wealth of long-term shareholders,[5] and managers will undertake an exchange when market conditions are favorable to achieve this end. The model, as shown in Figure 1, has three time points; $t = 0$, referring to the time prior to the exchange when market values of residual equity and liabilities are observed, $t = 1$, referring to the time when the exchange occurs and $t = 2$, referring to the future point at which the catastrophe bond is settled. Any assets remaining at that time are distributed to the shareholders and the firm ceases to exist.[6]

The firm's long-term shareholders are those who own shares at $t = 0$ and hold them to $t = 2$. There may be other shareholders at $t = 0$ and $t = 2$, but management will not have an incentive to consider their wealth objectives in the exchange; this suggests that the long-term shareholders may benefit at the expense of myopic traders.[7] Nor is management constrained to consider the wealth objectives of its other stakeholders, except to honor their contractual obligations and maintain regulatory compliance.[8]

[5]Long-term shareholders could include corporate insiders, but may also include external "buy-and-hold" investors such as institutional investors and passively managed equity funds.

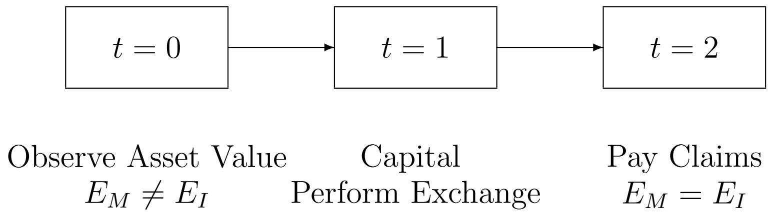
[6]Without loss of generality, the firm could continue perpetually with remaining assets distributed as dividends or reinvested in other firm assets.

[7]While the managers' fiduciary responsibilities extend to all shareholders, since traders rarely vote their shares, they are less engaged in corporate governance. Consistent with the findings of Grossman and Hart (1980), passive shareholders with small relative stakes have an incentive to free-ride and not vote their shares systematically, which in turn makes their wealth objectives more difficult to enforce.

[8]It is important to note the model's limits. Since this study analyzes optimal managerial behavior during a discrete period of time and assumes dissolution of the firm at the end of the timeline, the results do not analytically anticipate a long-lived firm with multiple financing opportunities and indefinite life. While our model is a one-shot game, we assume that information asymmetry is unimportant since parametric triggers are exogenous to managerial decision, but possible differences of opinion or different risk bearing capacities enable gains from trade. If this model were extended to an indefinite number of periods, the managers' or catastrophe bond investors' opinions may update and converge, eroding future gains from trade.

Figure 1: Capital Restructuring Model Timeline

Model timeline in which E_M is the market's estimate of the firm's equity value, given the aggregate market opinion or contribution to the aggregate investor portfolio. E_I is management's estimate of the firm's equity value, given their opinion about liability value or contribution to the firm's total business portfolio risk.



In our model, at $t = 0$, management has its own estimate of the probability distribution of claims, including the stochastic liability's contribution to portfolio volatility. Management also observes the required yields for catastrophe bonds and the market price of new obligations that they could assume, which reflect the market opinion of the loss distribution of such contingent claims. We assume that these new obligations would typically be naïve customers or employees: small-stakes, price-taking agents who will neither pay a premium nor demand a discount for leverage. Such stakeholders will engage with any firm that is not in obvious financial distress. Any difference of opinion or opportunity between management and market estimates of loss volatility is expressed as β ($\beta \neq 1$ represents some degree of difference of belief). With no discrepancy concerning volatility estimates, $\beta = 1$. If $\beta > 1$ ($\beta < 1$), then the market opinion of volatility is higher (lower) than management's opinion. Similarly, discrepancy between manager and market opinions of expected loss is expressed as γ. Note that differences regarding the liability volatility estimates will also affect the estimated covariance between the firm's asset returns and contingent liability value.

Observing such differences, management determines whether an opportunity exists to increase the wealth of long-term shareholders by issuing a catastrophe bond (i.e. laying off the risk at the cost of a certain payment and reducing liabilities) or increasing the contingent liability (i.e. taking on more similar obligations). Management also observes market values for the company's equity and contingent liability (i.e. the notional amount of a catastrophe bond necessary to fully fund the contingent liability). If management issues the catastrophe bond, the bond's investors would become a new class of "synthetic equity" holders of the firm (in that they own a residual claim based on the terminal value of the contingent liability only), and the firm has a new asset in the special purpose vehicle created to facilitate the bond's issue. This asset offsets the expected loss of the contingent liability at a rate of α $(0 < \alpha \leq 1)$ when the bond's investors assume a $1-\alpha$ proportion of the liability in exchange for their specialized equity stake. Conversely, management may increase the expected loss (by adding more risky obligations) at a rate of $1 + \alpha$ and using the revenues generated to retire equity. At $t = 1$, the capital restructuring occurs (losses increase or decrease by a factor of α) and at $t = 2$, all claims expire, obligations are paid to catastrophe bond investors and any remaining assets are distributed to the $t = 2$ shareholders.

We begin by noting that the contingent liability contributed to the firm's equity value in this way: the net value of the liquid assets supporting the claim less the expected value of the contingent liability. This value remains constant until the random outcome is known at $t = 2$ and losses are paid out. Therefore, a change in the value of liabilities (caused by either incurring more obligations or laying off risk to the catastrophe bond investors) at $t = 1$ will require an equal and opposite change in the value of the equity. We assume that management will not overcompensate for the contingent liability, that is, reduce expected losses below 0 by selecting $\alpha > 1$. Note that the difference between the management's estimate of equity value and the market estimate of equity value may reflect either differences of opinion or opportunity regarding the probability distribution of the loss reserves and asset values.[9] The firm's volatility estimate reflects management's opinion about the expected value of the contingent liability, whereas the outside estimates reflect publicly available information and aggregate investor opinions. When these values differ, management may have an opportunity

[9] Following Myers and Majluf (1984), we assume that capital markets are efficient with respect to publicly available information, but that investors may have heterogeneous beliefs or idiosyncratic opportunity to bear risk; this implies that the current market value of the firm corresponds to the discounted value of its expected future cash flows, conditional on whatever publicly available information and estimates the market happens to possess about the firm. For more about heterogeneous beliefs in capital markets, see Levy, Levy, and Benita (2006).

to engage in an exchange that will benefit its long-term shareholders if the manager's beliefs are correct.[10] Thus, we assume that managers are acting in the best interest of their own shareholders and *as if* their beliefs are closest to reality. If the managers are wrong, they will make the wrong choice and shareholders may be negatively impacted.

We now consider the firm's partial balance sheet reflecting the assets and claims in question. We can define the firm's equity value as E_j in equation (1). Here, E is value of equity, A is value of tangible, liquid assets and L is the expected present value of the contingent liability. The subscript j is either M (market value) or I (insider value):

$$E_j = A_j - L_j. \tag{1}$$

If the firm's liquid assets are invested in an index fund that is fully disclosed to investors, they can be readily valued without dispute and market values equal insider values.[11] We are left with E_j sensitive to only L_j. When the estimates of expected losses differ, management's estimate of firm equity value is defined by equation (2):

$$E_I = A - L_I, \tag{2}$$

and the market value is defined by equation (3):

$$E_M = A - L_M. \tag{3}$$

Indeed, even if both managers and outsiders agree on underlying expected losses, there may be heterogeneous beliefs about loss variability, which could also impact assumptions

[10] Note that such an exchange does not necessarily represent an arbitrage opportunity, but an action based on different opinions regarding probability distributions and gains from trade based on those differences and differences in risk preferences and capacity for risk-bearing. Long-term shareholders may indeed lose money as a result of such an action, but management is acting based on their own beliefs (which may include inside information) and assumptions about the dynamics of the firm's contingent liability.

[11] If the issuer does not hold the market portfolio on the asset side, there may be additional opportunities for capital structure arbitrage that are not modeled here.

about covariance between assets and losses. A prospective customer's estimate of the firm's product values may depend on the firm's financial liquidity, especially if a risky liability (like a long-term product warranty) is considered a valuable component of the product purchase. Similarly, customer risk aversion may be a factor when customers consider their reservation prices.

In cases where outsider opinions about either expected loss or asset return volatility differs from management's opinions, management may consider making a debt-for-equity or equity-for-debt exchange to enhance the wealth of $t = 0$ shareholders who will hold their shares until $t = 2$.

3.1 Valuation When Assets and Liabilities are Risky

As shown by Rubinstein (1976) and Cox, Ross, and Rubinstein (1979), option pricing principles can be used to value projects and securities with or without limited liability. It has been suggested that these models require continuous trading and publicly tradable securities, based on no-arbitrage and replication arguments in a continuous time model. First, Rubinstein (1976) derives the option pricing model in a discrete time framework. Second, while it may not seem obvious at first, contingent liabilities are increasingly tradable. For insurers, there is a liquid reinsurance market that constantly enables risk trading. Moreover, our example relies upon a liquid market for catastrophe bonds, in which losses are bundled and sold to investors in exchange for low-default risk returns plus risk premium coupons. Thus, these constraints do not apply to the application of the option pricing framework in this context.[12] Furthermore, if the firm is investing at least some of its liquid assets in risky securities, the strike price for the exchange option is also a random variable and may be valued using the "preference-restricted" version of the Merton-Margrabe model (cf. Mer-

[12]In fact, tradability is not a strict requirement to apply the option pricing framework used here. Stapleton and Subrahmanyam (1984) show that standard no-arbitrage justifications are not necessary for applying models such as Merton-Margrabe to such analyses. Specifically, the functional form of the Merton-Margrabe model still obtains so long as one plausibly assumes that assets and liabilities are jointly lognormally distributed, *and* investors have constant relative risk aversion (CRRA) preferences.

ton (1973) and Margrabe (1978)). The Merton-Margrabe model shows that an option to exchange one risky asset for another can be valued according to the following formula:[13]

$$E_j = A_j N(d_1) - L_j N(d_2). \tag{4}$$

In equation (4), E_j is value of equity, A_j is current value of assets and L_j is expected value of the contingent liability. The function $N(\bullet)$ is a draw from the standard normal distribution function. Additional parameters of the Merton-Margrabe Model are defined below:

$$d_{1j} = \frac{ln\left(\frac{A_j}{L_j}\right) + 0.5V_j^2 t}{V_j\sqrt{t}}, d_{2j} = d_{1j} - V_j\sqrt{t}, \text{ and}$$

$$V_j^2 = \sigma_{jA}^2 + \sigma_{jL}^2 - 2\sigma_{jAL},$$

where j takes the value of M for value of equity in the opinion of outsiders and I for value of equity in the opinion of the managers, σ_{jk} is the standard deviation of the variable k (either A for assets or L for losses) under j, σ_{jAL} is the covariance between assets A and liabilities L under j, and t is time to bond maturity. Since β and γ reflect degrees of difference in opinion or opportunity, outsider estimates of pre-exchange valuation inputs for d_{1M}, d_{2M}, and V_M^2 would be:

$$d_{1M} = \frac{ln\left(\frac{A}{\gamma L}\right) + 0.5V_M^2 t}{V_M\sqrt{t}}, d_{2M} = d_{1M} - V_M\sqrt{t},$$

and

$$V_M^2 = \sigma_A^2 + \beta^2\sigma_L^2 - 2\beta\sigma_{AL}.$$

Note that V_j^2 is the variance (estimated independently by management and investors) of the portfolio of long assets and short liabilities, and for simplicity, probability distributions for

[13] Fischer (1978) derives a model similar to Merton (1973) and applies it to the valuation of indexed bonds. Catastrophe bonds are essentially indexed bonds, with the final payoff being indexed to the bond's indicated trigger.

assets and liabilities are stable over time.

3.2 The Exchange

When management chooses to undertake an exchange, they will essentially increase (reduce) the expected value of the loss reserves by some proportion α. When $0 < \alpha < 1$, management is issuing a catastrophe bond, essentially making the bond's investors a new class of equity holders, whose residual claim is limited to the performance of the catastrophe bond. When $\alpha > 1$, management generates cash by increasing contingent liability claims and using the proceeds to retire equity. Thus, managers believe that the post-exchange value of the firm is adjusted by α, outsiders will evaluate the change in value based on their estimates of β and γ. In the context of equation (4), we propose lemma 1.

Lemma 1. *The post-exchange value of equity is a decreasing function of α.*

Proof. The relationship between post-exchange value of equity and α is determined by taking the partial derivative of equity value:

$$\frac{\partial E}{\partial \alpha} = A \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial \alpha} - \gamma L \left[N(d_2) \right] - \alpha \gamma L \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial \alpha}. \tag{5}$$

Since

$$d_1 = \frac{\ln \frac{A}{\alpha \gamma L} + 0.5 V^2 t}{V \sqrt{t}}, \quad \frac{\partial N(d_1)}{\partial d_1} = n(d_1), \frac{\partial N(d_2)}{\partial d_2} = n(d_2),$$

and

$$\frac{\partial d_1}{\partial \alpha} = \frac{\partial d_2}{\partial \alpha} = -\frac{1}{\alpha V \sqrt{t}},$$

it follows that

$$d_1 V \sqrt{t} = \ln \frac{A}{\alpha \gamma L} + 0.5 V^2 t, \; d_1 V \sqrt{t} - 0.5 V^2 t = \ln \frac{A}{\alpha \gamma L},$$

and

$$A = \alpha \gamma L e^{d_1 V \sqrt{t} - 0.5 V^2 t}. \tag{6}$$

Therefore,

$$\frac{\partial E}{\partial \alpha} = A\frac{n\left(d_1\right)}{V\sqrt{t}} - \alpha\gamma L \cdot N\left(d_2\right). \tag{7}$$

Substituting the right-hand side of equation (6) for A in equation (7) and expanding the $n(\bullet)$ operator, we obtain

$$\begin{aligned}
\frac{\partial E}{\partial \alpha} &= -\frac{\alpha\gamma L}{\alpha V\sqrt{2t\pi}} e^{d_1 V\sqrt{t} - 0.5V^2 t} e^{-0.5 d_1^2} - \frac{\gamma L}{V\sqrt{2t\pi}} e^{-0.5\left(d_1 - V\sqrt{t}\right)^2} - \alpha\gamma L \cdot N\left(d_2\right) \\
&= -\frac{\gamma L}{V\sqrt{2t\pi}} \left(e^{-0.5\left(d_1 - V\sqrt{t}\right)^2} - e^{-0.5\left(d_1 - V\sqrt{t}\right)^2}\right) - \alpha\gamma L \cdot N\left(d_2\right) \\
&= -\alpha\gamma L \cdot N\left(d_2\right). \qquad (8)
\end{aligned}$$

Thus, as long as liabilities are positive, equity value decreases in proportion to liabilities changed and any deviation between manager and outsider beliefs about expected loss.

□

Lemma 2. *The post-exchange value of equity is an increasing function of β for firms with low risk-neutral default probabilities.*

Proof. The relationship between post-exchange value of equity and β is determined by taking the partial derivative of equity value:

$$\frac{\partial E}{\partial \beta} = A\frac{\partial N\left(d_1\right)}{\partial d_1}\frac{\partial d_1}{\partial \beta} - \alpha\gamma L\frac{\partial N\left(d_2\right)}{\partial d_2}\frac{\partial d_2}{\partial \beta}.$$

Using definitions from the previous proof, as well as the observations that:

$$\frac{\partial d_1}{\partial \beta} = -w_L d_2 \text{ and } \frac{\partial d_2}{\partial \beta} = -w_L d_1,$$

and similar steps used to derive equation (8), we can show that:

$$\begin{aligned}
\frac{\partial E}{\partial \beta} &= A\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \beta} - \alpha\gamma L\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial \beta} \\
&= -A\cdot n(d_1)\, w_L d_2 + \alpha\gamma L\cdot n(d_2)\, w_L d_1 \\
&= \frac{-\alpha\gamma L e^{d_1 V\sqrt{t} - .5V^2 t} e^{-0.5 d_1^2} w_L d_2 + \alpha\gamma L e^{-0.5 d_2^2} w_L d_1}{\sqrt{2\pi}} \\
&= \alpha\gamma L w_L n(d_2)\left[d_1 - d_2\right] \qquad (9)
\end{aligned}$$

As long as $w_L > 0$, increasing the trader's opinion of loss volatility increases the trader's implied value of equity. □

Two examples follow in which expected management estimates of volatility and expected value of liabilities differs from outsider estimates.

If management believes that converting a contingent liability to a catastrophe bond will add value for the firm, they may be able to increase value for long-term shareholders by issuing that bond and bringing the bond's investors as a new class of "synthetic equity" holders of the firm. Alternatively, management could increase value for long-term shareholders by increasing its contingent liability value (for example, selling more products with long-lived warranties), using the proceeds to reduce outstanding equity and thereby increase leverage. Management need not retire equity to make such an exchange worthwhile; they could instead use the new cash flow to finance new projects or increase capital levels to satisfy bond rating agencies.

In order to test the impacts of capital structure shifts, we assume that the nature of these transactions limits signaling frictions (for example, issuing a catastrophe bond or increasing contingent obligations) and that the market will become aware of the terminal value of the liability at $t = 2$.[14] Benefits accruing to long-term shareholders would decline if signaling frictions were present.[15] At $t = 1$, management seeks to maximize shareholder wealth given

[14]Becoming aware of the outcome does not indicate that either management or the market actually knew the *a priori* loss distribution.

[15]Signalling frictions could include moral hazard, credit risk, or other information from which an investor

expectations of $t = 2$ outcomes. Since all outcomes will become known at $t = 2$, the market value and the insider value will be equal, $E_{M2} = E_{I2}$, so we suppress the j subscript for expected values at $t = 2$.

The definition of terminal long-term shareholder wealth depends on the type of exchange. When the firm issues a catastrophe bond, its investors become a new class of equity holders, effectively diluting long-term shareholders' wealth. Long-term shareholders' diluted ownership interest proportion is the ratio of market value of equity at $t = 0$ to market value of equity at $t = 1$. Their overall change in wealth is the diluted value of their equity after the exchange less the value of equity prior to the exchange, given as Ψ_D in equation (10):

$$\Psi_D = \frac{E_{M0}}{E_{M1}} E_2 - E_{I0}. \tag{10}$$

Management should engage in the exchange only when diluted $t = 2$ wealth exceeds undiluted $t = 0$ wealth.

When the firm increases contingent liabilities and uses the proceeds to retire equity, the transaction is accretive to the long-term shareholders: after the transaction, they own a larger percentage of the firm's equity than they did prior to the exchange. The initial wealth of long-term shareholders is the ratio of market value of equity at $t = 1$ to the market value of equity at $t = 0$, multiplied by the $t = 0$ insider (subscript I) value of the firm's equity, and the change in wealth, given as Ψ_A in equation (11):

$$\Psi_A = E_2 - \frac{E_{M1}}{E_{M0}} E_{I0}. \tag{11}$$

Equation (11) represents the value of their portion of the firm's equity prior to the exchange less the proportion of total equity market value after the exchange, and reflects the change in

might infer implicit disclosure of information. By using a parametric trigger catastrophe bond, we abstract away most signalling frictions for the purpose of this analysis. It would be difficult to measure the impact of signalling frictions; however, if they are present, they would likely be priced into the required yield for the equity-for-debt or debt-for-equity exchange and reduce the long-term shareholder benefit from such an exchange.

value of the long-term shareholders' equity. Again, management will engage in the exchange only when the $t = 2$ long-term shareholders' wealth exceeds the $t = 0$ wealth.

For a firm issuing a catastrophe bond, the change in long-term shareholder wealth as a function of expected value of the liability is:

$$\begin{aligned}\frac{\partial \Psi_D}{\partial \alpha} &= E_{M0}\frac{\partial E_M^{-1}}{\partial \alpha}E_2 + E_{M0}E_{M1}^{-1}\frac{\partial E_2}{\partial \alpha} \\ &= \frac{E_{M0}E_2}{E_{M1}^2}\alpha\gamma L \cdot N\left(d_2\right) - \frac{E_{M0}}{E_{M1}}\alpha L \cdot N\left(d_2\right) \\ &= \frac{E_{M0}}{E_{M1}}\alpha L \cdot N\left(d_2\right)\left[\gamma\frac{E_2}{E_{M1}} - 1\right]. \end{aligned} \tag{12}$$

The unsimplified version of equation (12) provides the intuition; the first term on the right-hand side represents the change in value of the long-term shareholders' equity value for each unit change in expected liability value (given traders' difference of opinion or opportunity), whereas the second term represents the cost or benefit (dilution or accretion) resulting from the catastrophe bond issue. If a value of α can be found that sets makes these two terms are equal, there is an optimal exchange. The second version facilitates signing the derivative and predicting the change. The ratio outside the square brackets is strictly not negative since, as option values, neither the numerator nor denominator of the fraction can be negative. Inside the brackets, $\gamma >= 0$ by definition and both numerator and denominator of the fraction are non-negative. The sign of equation (12) depends on the ratio of terminal ($t = 2$) value of equity to the market value of equity immediately after the exchange ($t = 1$), scaled by the traders' level of difference of opinion or opportunity regarding expected liability value (γ).

When retiring equity, we take the partial derivative of equation (11) with respect to α:

$$\begin{aligned}\frac{\partial \Psi_A}{\partial \alpha} &= \frac{\partial E_2}{\partial \alpha} - \frac{\partial E_{M1}}{\partial \alpha}\frac{E_{I0}}{E_{M0}} - E_{M1}\frac{\partial E_{M0}^{-1}}{\partial \alpha} - \frac{E_{M0}}{E_{M1}}\frac{\partial E_{I0}}{\partial \alpha} \\ &= -\alpha L \cdot N\left(d_2\right) + \alpha\gamma L \cdot N\left(d_2\right)\frac{E_{I0}}{E_{M0}} \\ &= \alpha L \cdot N\left(d_2\right)\left[\gamma\frac{E_{I0}}{E_{M0}} - 1\right] \end{aligned} \tag{13}$$

Again, the unsimplified version of equation (13) provides an intuitive explanation: the first term represents the change in the long-term shareholders' portion of the post-exchange equity. The second term represents the increase in shareholder wealth that results from reducing the number of shares outstanding.

Let us first consider, for example, the case in which outsiders believe that liability volatility is higher than managers do. We will refer to this deviation as $\beta > 1$, where $\beta = 1$ indicates no disagreement. Option valuation mechanics suggest that option value increases in volatility, so a higher market opinion of volatility indicates that equity is overvalued in the market and equity issue would be accretive. We now demonstrate this relationship with a proposition.

Proposition 1. *When outsiders believe that liability volatility is higher than managers do ($\beta > 1$), and managers issue equity a catastrophe bond (i.e., α decreases), long-term shareholder wealth will increase.*

Proof. To determine the sensitivity of long-term shareholder wealth changes resulting from equity issue to differences in volatility estimate, we find first-order conditions for how shareholder wealth changes with respect to α, proportion of contingent liability securitized and β, level of difference between management and outsider opinions of volatility or volatility contribution to each parties' portfolio.

Setting the first-order conditions equal to zero, a closed-form solution for α is available (given one equation and one unknown) but intractable for general purposes. The intractability arises from the fact that several of the partial derivatives contain functions of α that are not readily factored. However, the solution can be discovered using either linear programming or trial and error. Omitting the solvency constraint for explanation purposes, the managers will find a value of α that solves the equation (14), given their observation of implied information asymmetry to identify the values of β and γ:

$$\frac{E_{M0}E_2}{E_{M1}^2}\left(\frac{\partial E_{M1}}{\partial \alpha} + \frac{\partial E_{M1}}{\partial \beta}\right) = E_2\frac{\partial E_{M0}}{\partial \beta} + E_{M0}\frac{\partial E_2}{\partial \alpha} \tag{14}$$

Such a value of α will generate a benefit from issuing a catastrophe bond less the dilution associated with creating a new class of equity holders. Such an interior solution will exist when β is sufficiently higher than γ but both are greater than 1. $\square$

Now, let us consider the impact of retiring equity when the outsider opinion of loss volatility is higher than managers'.

Proposition 2. *When management believes that the market's opinion regarding liability volatility is higher than their own ($\beta > 1$), and increases contingent liabilities to retire equity (i.e., α increases), long-term shareholder wealth will decrease.*

Proof. To determine the sensitivity of long-term shareholder wealth from catastrophe bond issue to differences in volatility estimate, we take the cross-partial derivative of long-term shareholder wealth changes with respect to α, proportion of liability securitized and β, level of deviation between management and market opinions of volatility. To accomplish this, we will take the partial derivative of equation 13 with respect to β.

$$\begin{aligned}\frac{\partial^2\Psi_D}{\partial\alpha\partial\beta} &= \frac{\partial-\alpha L\cdot N\left(d_2\right)}{\partial\beta}+\frac{\partial\alpha\gamma L\cdot N\left(d_2\right)\frac{E_{I0}}{E_{M0}}}{\partial\beta}\\ &= \alpha L\cdot n\left(d_2\right)w_L d_1-\alpha\gamma L\cdot n\left(d_2\right)\frac{E_{I0}}{E_{M0}}-\alpha^2\gamma^2L^2N^2\left(d_2\right)\frac{E_{I0}}{E_{M0}^2} \qquad (15)\end{aligned}$$

$\square$

The counterfactual of Lemma 1 suggests that equity value falls when the firm retires equity using funds generated from issuing the catastrophe bond. This makes intuitive sense; when management believes that market estimates of loss volatility are higher than their own, the value of equity will be higher. Retiring overvalued equity is typically a poor use of shareholder funds, and would reduce long-term shareholder wealth.

We will forgo formal statement of propositions about the shareholder wealth effects of issuing and retiring equity when disagreements exist about expected liability values. The intuition is similarly straightforward. If managers believe that outsiders have overstated expected liability values, the price of reinsurance will be high and managers would not want to purchase it. Similarly, if potential customers are risk-averse, they may be willing to pay a premium for a product with an implicit or explicit long-lived warranty. In such cases, managers can inflate prices and sell more warrantied product, using the extra proceeds to retire equity.

Now, we will examine the inter-connections between sources of difference of opinion or opportunity. First, we will look at the change in equity value when both managers' opinion of loss and volatility differ from outsider beliefs.

Proposition 3. *For a solvent firm, the market value of equity will decline with increases in contingent liabilities when investor estimates of expected loss and volatility exceed those of managers.*

Proof. It can be shown that the third partial derivative of equity value with respect to change in expected losses (α), deviation of beliefs regarding expected losses and deviation of beliefs regarding loss volatility is:

$$\frac{\partial^3 E}{\partial\alpha\partial\beta\partial\gamma} = L \cdot w_L \cdot n\left(d_2\right)\left[d_1 - \frac{d_1 d_2}{V\sqrt{t}} - \frac{\sqrt{V}}{\sqrt{t}}\right], \tag{16}$$

where $w_L = \dfrac{\beta\sigma_L^2 - \sigma_{AL}}{V^2} < 0$,[16] which represents the weight of contingent liabilities in the firm's hedged portfolio that minimizes risk for the insurer, under the outsider estimates of liability volatility and management's estimate of covariance.[17]

The equity value decrease arises from a negative sign on the derivative shown above. Losses (L), time (t), volatility (V) and $n(d_2)$ are always positive. For a solvent firm ($A > L$), $d_2 > 1$, therefore, the bracketed term will be positive. Thus, the derivative's negative sign arises from the negative sign on w_L. □

[16] Here, we assume that assets and losses are positively correlated.

[17] According to Clairaut's Theorem, it does not matter which order we take multivariate partial derivatives. However, we achieve the same result when we calculate $\frac{\partial^3 E}{\partial\alpha\partial\gamma\partial\beta}$.

If increasing expected contingent liabilities destroys shareholder value, managers would optimally choose not to do so. However, it may be prudent to issue a catastrophe bond if shareholder wealth would be enhanced by such a move. The next proposition explores such a transaction.

Proposition 4. *For a solvent firm, market values of equity will increase upon issuance of a catastrophe bond.*

The proof for this proposition follows from the previous proof. However, when considering long-term shareholder wealth impacts, we must be concerned about the dilution effects of issuing a catastrophe bond. Referring to equation 12, recall that the first term represents the change in market value of equity resulting from the exchange, whereas the second term represents the dilution effect. Therefore, long-term shareholder wealth may be improved by such a transaction, as long as the benefits of a favorable catastrophe bond are not overwhelmed by the dilution effect. A series of numerical examples is presented in the next section to illustrate long-term shareholder wealth changes resulting from a series of possible transactions.

4 Numerical Examples

In this section, we analyze capital structure adjustments based on management beliefs (which may differ from outsider beliefs, even though the facts are not in dispute) about the value of the firm's assets, liabilities and the covariance between the two. These results suggest potentially wealth-enhancing transactions. We establish a base case in which management beliefs agree with outside beliefs and a catastrophe bond will neither add nor remove value relative to the firm retaining the risky liability. We then allow outsider beliefs to deviate from manager beliefs, showing a potential value-maximizing action. In each case, we assume for simplicity that the firm's asset values and volatility are exogenous and known to all. For our examples, we assume that $\sigma = 0.19$, which is a long-run estimate of the standard deviation of annual returns on a representative market portfolio, as calculated from the data available on Ken French's website (French, 2022).[18]

4.1 Base Case

First, we consider a case where there is no difference of opinion between management and outsiders;[19] i.e., they agree about the risk and value of assets and losses, and the covariance between assets and losses. Initially, we assume asset holdings (consisting of investments in the market portfolio, $\sigma = 0.19$) of \$1,500, and a portfolio of insurance policies with an expected loss of \$1,000 and $\sigma = 0.4$. In this case, management may issue a \$500 catastrophe bond, effectively reducing the firm's expected loss portfolio by \$500, without affecting the value of the firm's assets. Alternatively, the firm may sell products that create \$200 in new contingent liabilities and use the cash to retire equity, again without affecting the value of assets. Assuming no taxes or market frictions for simplicity, we show in Table 1 that shareholder wealth is not affected by leverage choices. These results are consistent with the predictions of Modigliani and Miller (1958) that in the absence of friction (direct or indirect) costs, the funding source is irrelevant to shareholder value.[20]

[18] Since the volatility of assets enters into the valuation equation in the V^2 term, it is important to use a reasonable estimate for the risk of the firm's assets. If the firm holds treasury securities, the risk of the asset portfolio would go away and the model would simplify to a simple Black-Scholes put option held by the policyholders. If the firm holds a specialized portfolio of risky securities, the wealth change will be related to the relative risk of the assets and losses as well as their covariance. However, since none of our exchange examples include a change in asset holdings, nor a difference of opinion about the riskiness of the asset pool, the impact of our asset standard deviation parameter choice should be minimal.

[19] Here, we assume that outsiders have homogeneous information with each other and managers, but differ with managers about what that information means.

[20] Note that these results rely on the strong assumptions of Modigliani and Miller (1958) and Modigliani and Miller (1963), namely that there are no direct or indirect "frictional" costs, individuals and corporations can borrow and lend at the same rate, and investors and management have access to the same information. Here, we assume that all parties have access to the same information, but derive different beliefs about the future from that information, or have idiosyncratic opportunities to bear risk.

Table 1: No Disagreement about Expected Loss or Volatility

Changes in insider shareholder wealth following recapitalizations of risky liabilities. Calculated using Merton-Margrabe model. The first model shows the capital structure of the firm and equity and expected liability value beliefs held by both management (insider) and outside investors (market). The second model shows the change in shareholder wealth resulting from issuing \$500 in equity at the market per-share price and using the proceeds to issue a catastrophe bond. The third model shows the change in shareholder wealth resulting from increasing risky liabilities by \$200 at the market price and using the proceeds to retire equity at the per-share market price.

	Base Case		Issue Equity		Retire Equity	
	Mgr.	Trader ($\beta = 1$)	Mgr.	Trader ($\beta = 1$)	Mgr.	Trader ($\beta = 1$)
Asset Value	\$1,500.00	\$1,500.00	\$1,500.00	\$1,500.00	\$1,500.00	\$1,500.00
Expected Loss	1,000.00	1,000.00	500.00	500.00	1,200.00	1,200.00
Asset Volatility	19%	19%	19%	19%	19%	19%
Loss Volatility	40%	40%	40%	40%	40%	40%
Time	1	1	1	1	1	1
Asset/Loss Covariance	0.152%	0.152%	0.152%	0.152%	0.152%	0.152%
Equity Value	\$551.06	\$551.06	\$1,000.75	\$1,000.75	\$413.77	\$413.77
Loss Value	948.94	948.94	499.25	499.25	1,086.23	1,086.23
Long-term portion of initial equity			55.06%			
Long-term portion of ending equity					75.09%	
Change in long-term wealth			\$0.00		\$0.00	

4.2 Volatility Estimates Differ (Market Volatility Higher)

In Table 2, we consider two exchanges in which management believes that laying off risky liabilities would add value for the firm because of heterogeneous beliefs or idiosyncratic opportunity about volatility. This is reasonable when management beliefs reflect lower volatility of the risky obligation than the outsider beliefs, or the managers' risk-bearing appetite is lower than outsiders, who assume a higher volatility of liabilities than managers do. In the first case, management engages catastrophe bond investors to contribute \$500 to a catastrophe bond that reverses the risky liability. In the second case, management increases the expected value of the risky liability by \$200 and uses the cash generated to retire equity.[21] In either case, the capital structure has changed, and shareholder wealth is impacted. Specifically, issuing a catastrophe bond increases the wealth of the long-term shareholders, whereas increasing risky liabilities and using proceeds to retire equity diminishes wealth. Management's objective is to maximize wealth for long-term shareholders (defined earlier as the pre-exchange shareholders who still hold shares after the exchange). Thus, the firm can increase the value of long-term shareholders' equity by issuing "synthetic equity" in the form of a catastrophe bond.[22]

4.3 Volatility Estimates Differ (Market Volatility Lower)

We now reverse the relative valuation estimates and show that when managers are able to raise cash in excess of expected liability value, they can use the proceeds to retire equity, holding assets constant.[23] As shown in Table 3, when management takes on more risky liabilities while increasing cash, they can use the cash to retire equity for the benefit of

[21] This could happen by selling more product backed by long-term warranties, by increasing pension obligations with deferred compensation, or selling an insurance policy whose payout depends on events outside the control of the firm.

[22] We acknowledge that these results largely mirror those of a similar exercise in O'Brien (2004). However, while that study allowed for differences of opinion about expected value of assets in the same direction, that result is not possible when all parties agree on the value and distribution of the asset pool.

[23] The ability to charge prices above expected liability stem from consumer risk aversion or difference of opinion about expected loss or volatility. For examples of such risk aversion, see Rabin and Thaler (2001).

Table 2: Volatility Estimates Differ (Market Volatility Higher)

Changes in long-term shareholder wealth following issuance of a catastrophe bond. Calculated using the Merton-Margrabe model. The first model shows the capital structure of the firm and equity, and expected risky liability value beliefs of both management (insider) and outsiders (market). The second model shows the change in shareholder wealth resulting from issuing "synthetic equity" in the form of a catastrophe bond. The third model shows the change in shareholder wealth resulting from increasing risky liabilities by $200 at the market price and using the proceeds to retire equity at the per-share market price.

	Volatility Estimates Differ		Issue Equity		Retire Equity	
	Mgr.	Trader ($\beta = 1.5$)	Mgr.	Trader ($\beta = 1.5$))	Mgr.	Trader ($\beta = 1.5$))
Asset Value	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00
Expected Loss	1,000.00	1,000.00	500.00	500.00	1,200.00	1,200.00
Asset Volatility	19%	19%	19%	19%	19%	19%
Loss Volatility	40%	60%	40%	60%	40%	60%
Time	1	1	1	1	1	1
Asset/Loss Covariance	0.152%	0.228%	0.152%	0.228%	0.152%	0.228%
Equity Value	$551.06	$616.30	$1,000.75	$1,008.33	$413.77	$501.67
Loss Value	948.94	883.70	499.25	491.67	1,086.23	998.33
Long-term portion of initial equity			61.12%			
Long-term portion of ending equity					81.4%	
Change in long-term wealth			$60.61		-$34.80	

long-term shareholders.

4.4 Differences Regarding Both Expected Liability Value and Volatility (Market Lower)

Up until now, we have assumed that management and outsiders differ only in their assumptions about volatility of the risky liability. We now introduce another layer of uncertainty: the expected value of the risky liability. In particular, we focus on cases in which that risky liability could be the value of a pension obligation, extended warranty, or similar risky obligation. When it comes to consumers and extended warranties, it is well-documented that many consumers exhibit *myopic risk aversion* and choose to purchase expensive extended warranties and rental car collision damage waivers (Rabin and Thaler, 2001). Similarly, employees often have skewed expectations about their own life expectancy, causing them to over- or under-estimate the volatility of their pension claim against their employer (Zhavoronkov, 2015).

For example, consider the exchanges similar to those examined earlier when managers believe that outsiders have different beliefs or idiosyncratic opportunities in which their estimates of both expected value and volatility are lower than their own, resulting in overvalued equity from the managers' point of view. Under conventional thinking, increasing equity typically signals management belief that equity while simultaneously diluting ownership. Past empirical work finds a corresponding drop in stock price usually results from such an issue. However, in this case, as shown in Table 4, we find that the indicated exchanges could improve long-terms shareholder wealth if the manager's beliefs turn out to be correct *no matter which direction the exchange is initiated.* In the next section, we will explore this finding as we examine figures illustrating the range of results of such exchanges.

Table 3: Loss Volatility Understated

Changes in long-term shareholder wealth following issuance of a catastrophe bond. Calculated using the Merton-Margrabe model. The first model shows the capital structure of the firm and equity and expected risky liability value beliefs of both management (insider) and outsiders (market). The second model shows the change in shareholder wealth resulting from issuing "synthetic equity" in the form of a catastrophe bond. The third model shows the change in shareholder wealth resulting from increasing risky liabilities by $200 at the market price and using the proceeds to retire equity at the per-share market price.

	Volatility Estimates Differ		Issue Equity		Retire Equity	
	Mgr.	Trader ($\beta = 0.75$)	Mgr.	Trader ($\beta = 0.75$)	Mgr.	Trader ($\beta = 0.75$)
Asset Value	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00
Expected Loss	1,000.00	1,000.00	500.00	500.00	1,200.00	1,200.00
Asset Volatility	19%	19%	19%	19%	19%	19%
Loss Volatility	40%	20%	40%	20%	40%	20%
Time	1	1	1	1	1	1
Asset/Loss Covariance	0.152%	0.076%	0.152%	0.076%	0.152%	0.076%
Equity Value	$551.06	$510.08	$1,000.75	$1,000.00	$413.77	$3423.50
Loss Value	948.94	989.92	499.25	500.00	1,086.23	1,157.50
Long-term portion of initial equity			51.01%			
Long-term portion of ending equity					67.15%	
Change in long-term wealth			-$40.60		$43.76	

Table 4: Differences Regarding Both Expected Value of Liability and Volatility (Market Lower)

Changes in long-term shareholder wealth following issuance of a catastrophe bond. Calculated using the Merton-Margrabe model. The first model shows the capital structure of the firm and equity and expected risky liability value beliefs of both management (insider) and outsiders (market). The second model shows the change in shareholder wealth resulting from issuing "synthetic equity" in the form of a catastrophe bond. The third model shows the change in shareholder wealth resulting from increasing risky liabilities by $200 at the market price and using the proceeds to retire equity at the per-share market price.

	Expected Loss and Volatility Lower		Issue Equity		Retire Equity	
	Mgr.	Trader ($\beta = 0.5, \gamma = 0.75$)	Mgr.	Trader ($\beta = 0.5, \gamma = 0.75$)	Mgr.	Trader ($\beta = 0.5, \gamma = 0.75$)
Asset Value	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00
Expected Loss	1,000.00	750.00	500.00	375.00	1,200.00	900.00
Asset Volatility	19%	19%	19%	19%	19%	19%
Loss Volatility	40%	20%	40%	20%	40%	20%
Time	1	1	1	1	1	1
Asset/Loss Covariance	0.152%	0.076%	0.152%	0.076%	0.152%	0.076%
Equity Value	$551.06	$750.51	$1,000.75	$1,125.00	$413.77	$603.76
Loss Value	948.94	749.49	499.25	375.00	1,086.23	896.24
Long-term portion of initial equity			66.71%			
Long-term portion of ending equity					80.45%	
Change in long-term wealth			$116.56		-$29.54	

4.5 Differences Regarding Both Expected Value of Liability and Volatility (Market Higher)

When managers believe that outsider opinions about the expected value of a risky liability exceed their own based on both expected value and volatility, they will observe that the company's market value is lower than their own estimate. When this happens as a result of overall market sentiment about risky liability distributions, we would expect that managers might raise more cash to support increasing the expected value of its risky obligations. In such cases, the conventional advice would be to exploit this difference of opinion by retiring "bargain-priced" equity or funding for new projects. However, in Table 5, we show two exchanges, both destroying shareholder value *in either direction*.

4.6 Exchange Example Charts

Figures 2 through 4 illustrate the range of possible long-term shareholder effects from the exchanges discussed in the prior section. In these figures, we hold all values constant except the manager and outsider beliefs about value of liabilities, preserving the spread between them when a deviation exists. The graphs show the change in shareholder wealth resulting from exchanges that involve issuing a catastrophe bond on the left, to exchanges that raise cash through operations (increasing expected value of risky liabilities) to retire equity on the right.

Figure 3 provides intuition for the counter-intuitive results shown in Table 4 (in which exchanges in either direction benefit long-term shareholders) and Table 5 (in which exchanges in either direction diminish shareholder wealth). In the latter graphs, we can see that the arbitrary exchanges chosen for the numerical examples resulted in either destroying or adding value no matter which exchange was executed. From Figure 3, we can see that interior extremes explain this interesting result. In the case where investor beliefs about both expected liability value and volatility are higher than managers', managers might increase long-term shareholder value (if managers are right) by issuing a catastrophe bond, but at

Table 5: Differences Regarding Both Expected Loss and Volatility (Market Higher)

Changes in long-term shareholder wealth following issuance of a catastrophe bond. Calculated using the Merton-Margrabe model. The first model shows the capital structure of the firm and equity and expected risky liability value beliefs of both management (insider) and outsiders (market). The second model shows the change in shareholder wealth resulting from issuing "synthetic equity" in the form of a catastrophe bond. The third model shows the change in shareholder wealth resulting from increasing risky liabilities by $200 at the market price and using the proceeds to retire equity at the per-share market price.

	Expected Loss and Volatility Higher		Issue Equity		Retire Equity	
	Mgr.	Trader ($\beta = 1.5, \gamma = 1.1$)	Mgr.	Trader ($\beta = 1.5, \gamma = 1.1$)	Mgr.	Trader ($\beta = 1.5, \gamma = 1.1$)
Asset Value	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00
Expected Loss	1,000.00	1,100.00	500.00	550.00	1,200.00	1,320.00
Asset Volatility	19%	19%	19%	19%	19%	19%
Loss Volatility	40%	60%	40%	60%	40%	60%
Time	1	1	1	1	1	1
Asset/Loss Covariance	0.152%	0.228%	0.152%	0.228%	0.152%	0.228%
Equity Value	$551.06	$556.14	$1,000.75	$962.64	$413.77	$443.28
Loss Value	948.94	943.86	499.25	537.36	1,086.23	1,056.72
Long-term portion of initial equity			57.77%			
Long-term portion of ending equity					79.71%	
Change in long-term wealth			$27.09		-$25.46	

Table 6: Differences Regarding Both Expected Loss (Market Lower) and Volatility (Market Higher)

Changes in long-term shareholder wealth following issuance of a catastrophe bond. Calculated using the Merton-Margrabe model. The first model shows the capital structure of the firm and equity and expected risky liability value beliefs of both management (insider) and outsiders (market). The second model shows the change in shareholder wealth resulting from issuing "synthetic equity" in the form of a catastrophe bond. The third model shows the change in shareholder wealth resulting from increasing risky liabilities by $200 at the market price and using the proceeds to retire equity at the per-share market price.

	Expected Loss Lower and Volatility Higher		Issue Equity		Retire Equity	
	Mgr.	Trader ($\beta = 1.5, \gamma = 0.75$)	Mgr.	Trader ($\beta = 1.5, \gamma = 0.75$)	Mgr.	Trader ($\beta = 1.5, \gamma = 0.75$)
Asset Value	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00
Expected Loss	1,000.00	750.00	500.00	375.00	1,200.00	900.00
Asset Volatility	19%	19%	19%	19%	19%	19%
Loss Volatility	40%	60%	40%	60%	40%	60%
Time	1	1	1	1	1	1
Asset/Loss Covariance	0.152%	0.228%	0.152%	0.228%	0.152%	0.228%
Equity Value	$551.06	$793.43	$1,000.750	$1,127.11	$413.77	$682.48
Loss Value	948.94	706.579	499.25	372.89	1,086.23	817.52
Long-term portion of initial equity			70.40%			
Long-term portion of ending equity					86.02%	
Change in long-term wealth			$153.42		-$60.23	

Table 7: Differences Regarding Both Expected Value of Liability (Market Higher) and Volatility (Market Lower)

Changes in long-term shareholder wealth following issuance of a catastrophe bond. Calculated using the Merton-Margrabe model. The first model shows the capital structure of the firm and equity and expected risky liability value beliefs of both management (insider) and outsiders (market). The second model shows the change in shareholder wealth resulting from issuing "synthetic equity" in the form of a catastrophe bond. The third model shows the change in shareholder wealth resulting from increasing risky liabilities by $200 at the market price and using the proceeds to retire equity at the per-share market price.

	Expected Loss Higher and Volatility Lower		Issue Equity		Retire Equity	
	Mgr.	Trader ($\beta = 0.5, \gamma = 1.5$)	Mgr.	Trader ($\beta = 0.5, \gamma = 1.5$)	Mgr.	Trader ($\beta = 0.5, \gamma = 1.5$)
Asset Value	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00	$1,500.00
Expected Loss	1,000.00	1,500.00	500.00	750.00	1,200.00	1,800.00
Asset Volatility	19%	19%	19%	19%	19%	19%
Loss Volatility	40%	20%	40%	20%	40%	20%
Time	1	1	1	1	1	1
Asset/Loss Covariance	0.152%	0.076%	0.152%	0.076%	0.152%	0.076%
Equity Value	$551.06	$162.92	$1,000.75	$751.51	$43.77	$67.36
Loss Value	948.94	1,337.08	499.25	749.49	1,086.23	1,432.64
Long-term portion of initial equity			21.71%			
Long-term portion of ending equity					41.34%	
Change in long-term wealth			-$333.82		$185.94	

Table 8: Comparative Statics

Comparative statics showing the predicted optimal exchange given managers' beliefs about expected value of liability and volatility estimates relative to outsiders. "Market higher" means that compared with their own expected value or volatility estimates, managers believe outsiders have overstated the expected value or volatility. "No difference" means that the managers' beliefs agree with outsider estimates. "Market lower" means that managers believe that outsiders' estimates of loss and volatility are too low.

	Expected Loss:		
	Market Higher ($\gamma > 1$)	No Difference ($\gamma = 1$)	Market Lower ($\gamma < 1$)
Volatility:			
Market Higher ($\beta > 1$)	Issue equity in some cases	Always issue equity	Always issue equity
No Difference ($\beta = 1$)	Always retire equity	Do nothing	Always issue equity
Market Lower ($\beta < 1$)	Always retire equity	Always retire equity	Either action will add value

some point, the dilution from issuing "synthetic equity" eliminates the gains from a low price to lay off risk and long-term shareholder value declines.

In Figures 5 and 6, we illustrate how the potential wealth transfers vary over the range of potential levels of disagreement. Figure 5 shows wealth transfers available when outsiders believe that losses will end up lower than managers do. Regions shaded in gray represent wealth transfers to long-term shareholders while regions shaded in black represent long-term shareholder wealth losses. We can see that when traders believe that losses will be lower than managers do ($\gamma < 1$), the best choice is to issue a catastrophe bond ($\alpha < 1$). The benefit to long-term shareholders is intensified when outsiders believe that loss volatility is lower than managers do ($\beta > 1$).

Moving to Figure 6, we see a different situation. Here, outsiders believe that losses will end up higher than managers do ($\gamma = 1.25$). In the south corner of the chart, we can see that the region over which shareholder value increases is represented by increasing risky liabilities and using the proceeds to retire equity ($\alpha > 1$). Note that there are also value-enhancing exchanges that issuing catastrophe bonds, but any exchange above a certain level of disagreement about volatility in this direction will destroy value for long-term shareholders. This is seen most prominently in the east corner of the chart where the value surface drops below the zero wealth transfer plane.

5 Conclusion

Prior research has shown that in most cases, firms that issue equity do so at the expense of the long-term shareholders. Our results, however, show situations in which differences of opinion or idiosyncratic risk-bearing opportunities may make equity issues value-enhancing for shareholders, even when there is no information asymmetry. Further research has shown that shareholders often gain advantages from transferring risk, including comparative advantages in risk-bearing and provision of real services. Our model shows new examples of how

Figure 2: Volatility Deviations

Examples of long-term shareholder wealth changes following exchanges, if manager beliefs are correct. The solid line corresponds with the exchanges illustrated in Table 2 and the dashed line corresponds with the exchanges illustrated in Table 3. Points on the left side of the graph correspond with exchanges in which the firm issues equity and uses the proceeds to purchase reinsurance. Points on the right side of the graph correspond with exchanges in which the firm increases risky liabilities and uses the proceeds to retire equity.

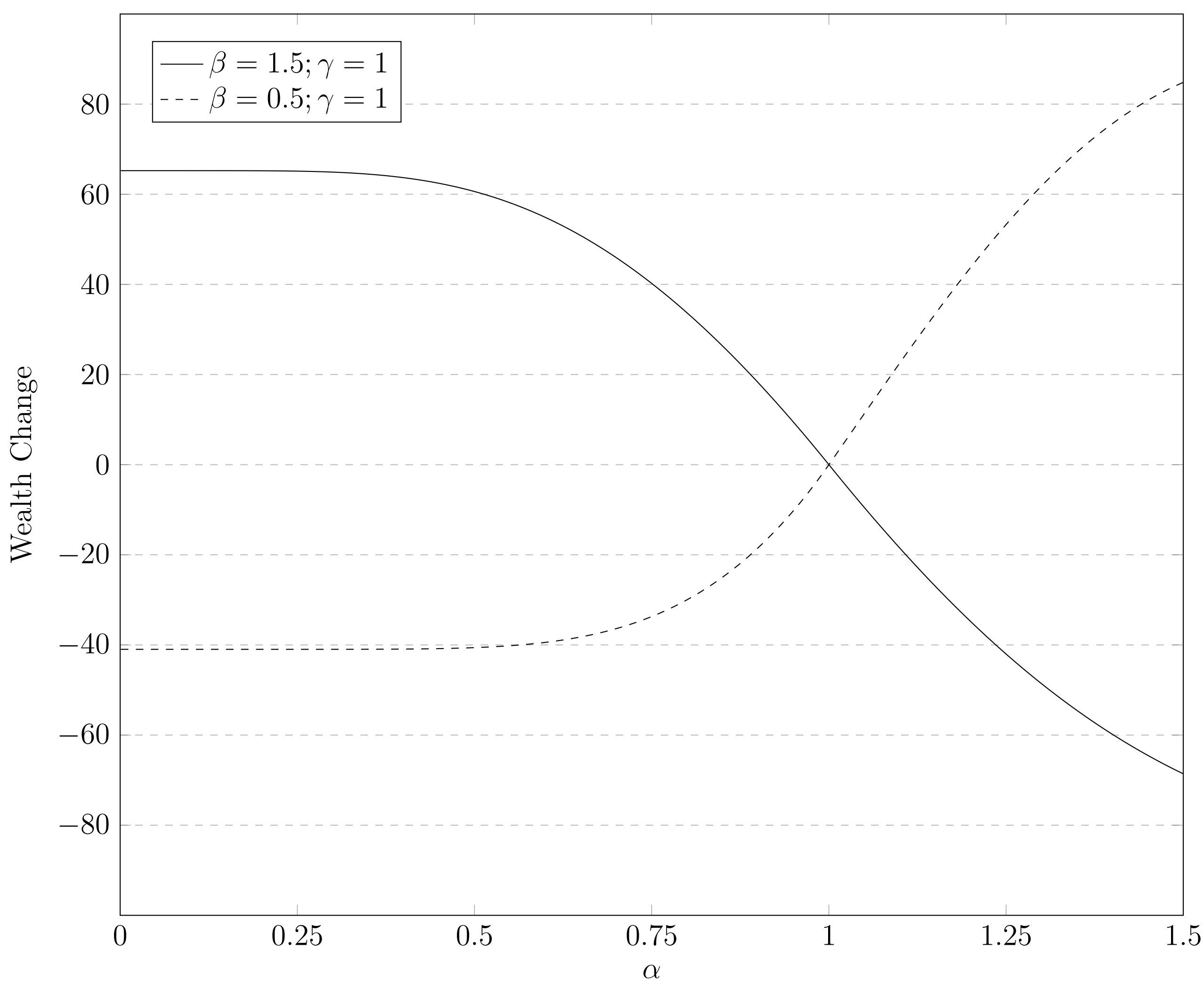


Figure 3: Expected Value of Liability and Volatility Deviations – Same Direction

Examples of long-term shareholder wealth changes following exchanges, if manager beliefs are correct. The solid line corresponds with the exchanges illustrated in Table 4 and the dashed line corresponds with the exchanges illustrated in Table 5. Points on the left side of the graph correspond with exchanges in which the firm issues equity and uses the proceeds to purchase reinsurance. Points on the right side of the graph correspond with exchanges in which the firm increases risky liabilities and uses the proceeds to retire equity.

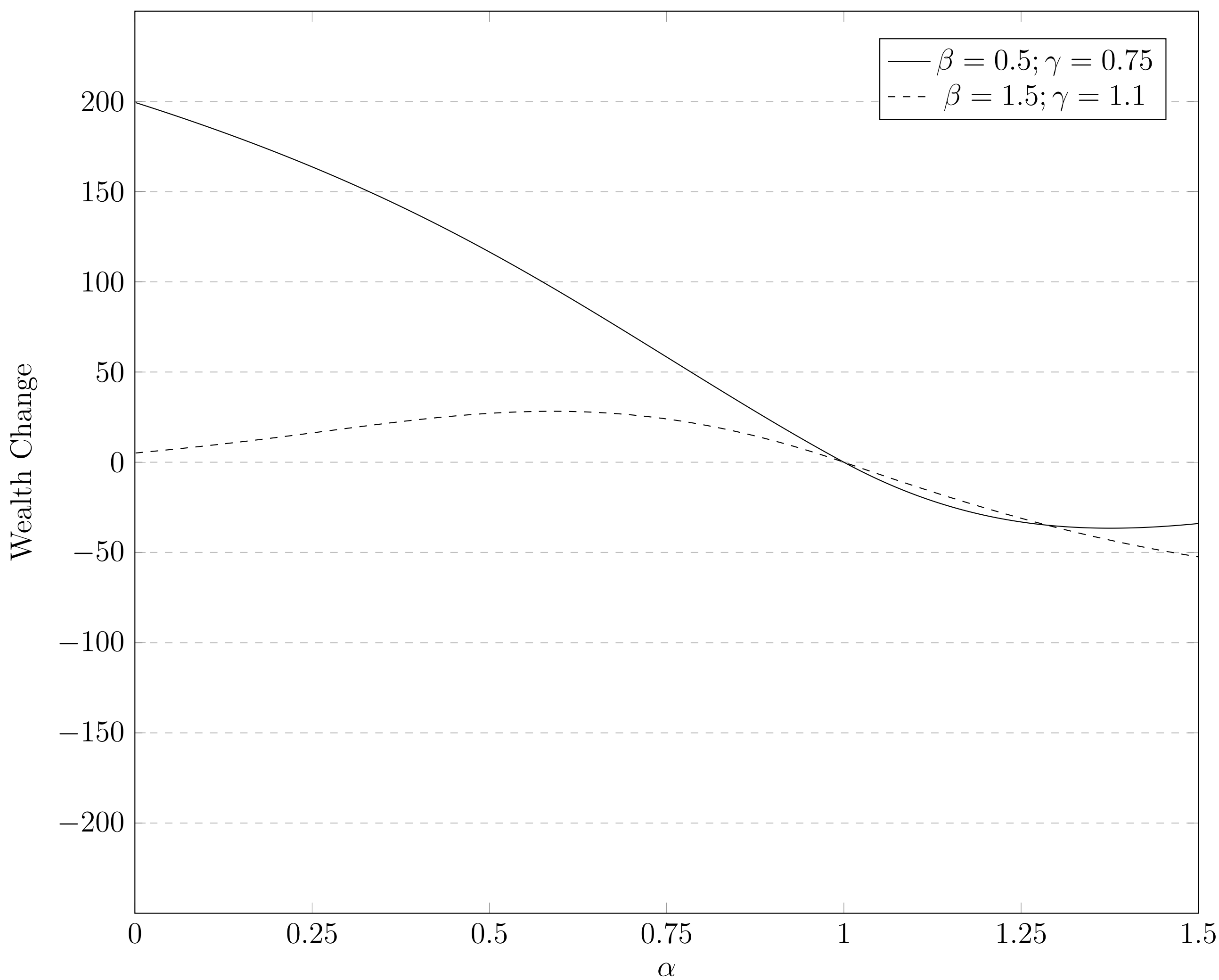


Figure 4: Expected Value of Liability and Volatility Deviations – Different Directions
Examples of long-term shareholder wealth changes following exchanges. The implied plane where wealth change equals zero represents no wealth transfer. The uneven surface expands the analysis from tables 4 and 5, showing wealth changes at varying levels of β while holding γ constant at 0.75. Points on the left side of the graph ($\alpha < 1$) correspond with exchanges in which the firm issues a catastrophe bond. Points on the right side of the graph ($\alpha > 1$) correspond with exchanges in which the firm increases risky liabilities and retires equity. Points colored gray indicate a transfer of wealth to the long-term shareholders, while points colored in black indicate loss of long-term shareholder wealth.

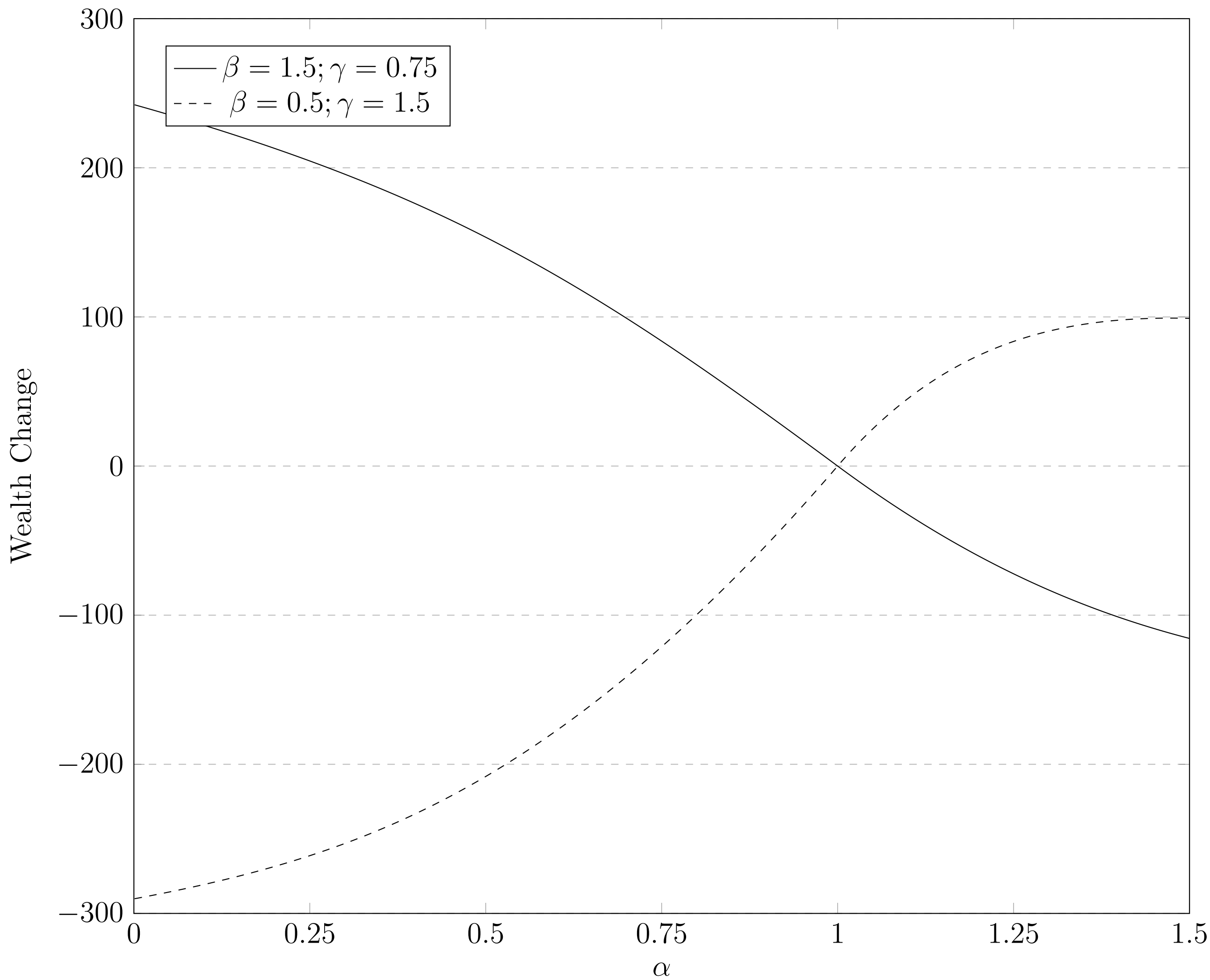


Figure 5: Expected Loss and Volatility Deviations – Same Direction

Examples of long-term shareholder wealth changes following exchanges. The implied plane where wealth change equals zero represents no wealth transfer. The uneven surface expands the analysis from tables 4 and 5, showing wealth changes at varying levels of β while holding γ constant at 0.75. Points on the left side of the graph ($\alpha < 1$) correspond with exchanges in which the firm issues a catastrophe bond. Points on the right side of the graph ($\alpha > 1$) correspond with exchanges in which the firm increases risky liabilities and retires equity. Points colored gray indicate a transfer of wealth to the long-term shareholders, while points colored in black indicate loss of long-term shareholder wealth.

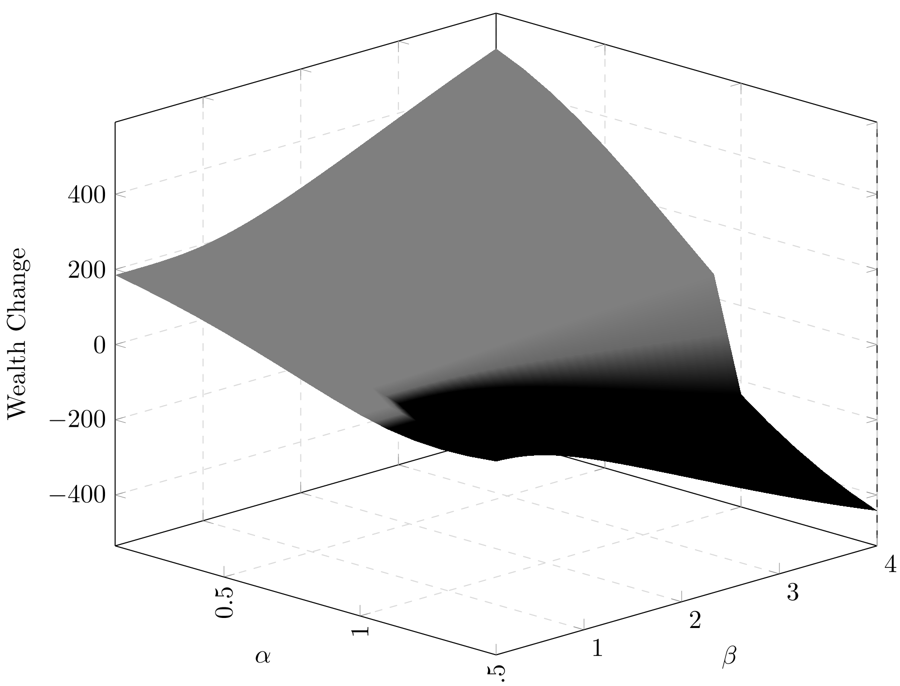
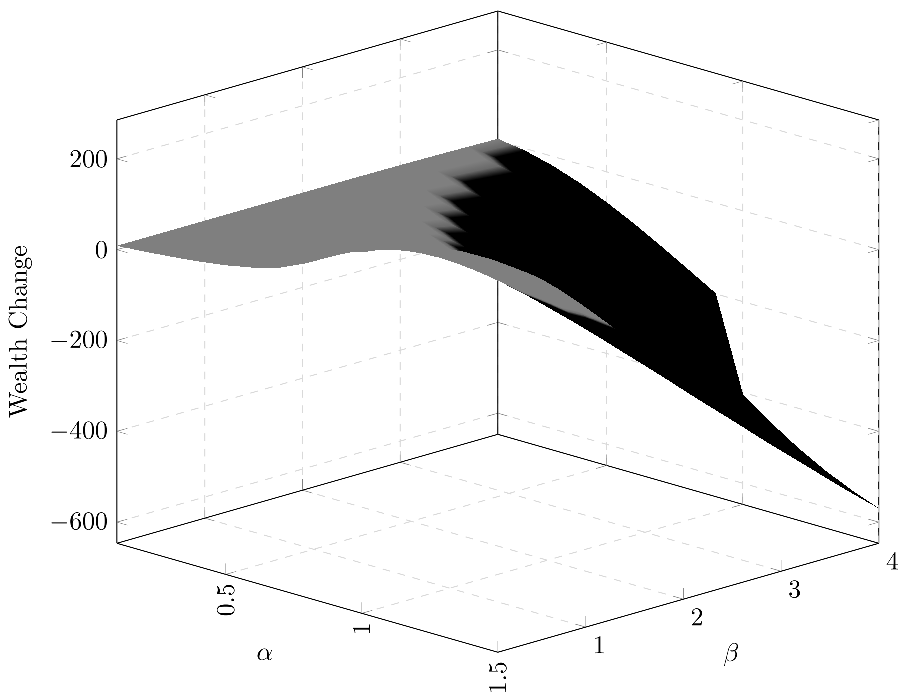


Figure 6: Expected Loss and Volatility Deviations – Same Direction

Examples of long-term shareholder wealth changes following exchanges. The surface expands the analysis from tables 4 and 5, showing wealth changes at varying levels of β while holding γ constant at 1.5. Points on the north corner of the graph ($\alpha < 1$ and $\beta > 1$, colored gray) correspond to situations in which outsiders believe that loss volatility is higher than do managers, and when the firm issues a catastrophe bond. Points on the south and east corners of the graph ($\alpha > 1$ and $\beta < 1$, darker in color) correspond with exchanges in which traders believe that loss volatility is lower than do managers, and when the firm increases risky liabilities and uses the proceeds to retire equity. Points colored gray indicate a transfer of wealth to the long-term shareholders, while points colored in black indicate loss of long-term shareholder wealth.



those comparative advantages can be harnessed for shareholders. When the managers can exploit risk aversion of potential claimants by charging higher prices when taking on risky obligations, they can increase the firm's leverage and use the proceeds to retire equity or invest in new projects. At the same time, if managers can add value by issuing "synthetic equity" in the form of a catastrophe bond, such a transaction may also be value-additive.

In this article, we show that firms with risky liabilities have unique opportunities to engage in capital structure arbitrage by executing transactions that take advantage of market conditions and frictions. We use the Merton-Margrabe option pricing model to analyze the impact of an exchange of risky assets. Contrary to the conventional rule, we show that long-term shareholders' wealth can increase when management engages in some exchanges, issuing catastrophe bonds. Finally, we show that an interior solution exists in some cases to maximize long-term shareholder value when both equity and liabilities are valued differently by myopic outsiders and potential claimants.

These findings provide new rationale for the findings of Myers and Majluf (1984). Our model shows how heterogeneous beliefs, differences in risk aversion, cost of real services, and covariance between asset and loss portfolios can generate unique opportunities for capital structure exchanges that are available to firms of all types, when heterogeneous beliefs exist between managers and outsiders.

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